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Multi-criteria Methods Applied in the Selection of a Brake Disc Material

Métodos multicriterio aplicados en la selección de un material para discos de freno

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Abstract

The selection of material for an automotive component is a complex process, because it involves an exploration of the main criteria according to the properties required by the component to be designed. The purpose of this study is to evaluate an alternative material in the manufacture of a brake disc in light SUV type vehicles, using multi-criteria methods; five candidate materials are taken into consideration for the desired application (Ti_6Al_4V, $Al_{10}Si$ C, AISI 304L, ASTM A 536 and ASTM A48). The multi-criteria methods (MCDM) used are: VIKOR - multidisciplinary optimization and compromise solution; ELECTRE I - elimination and options that reflect reality; COPRAS - proportional complex evaluation; ARAS - additive ratio evaluation; MOORA - multi-objective optimization based on radius analysis and the ENTROPIA method used for the weighting of criteria. It is concluded that the best alternative is the ASTM A536 material according to the COPRAS, ELECTRE I, and ARAS methods due to its low density, a high elastic limit and a good resistance to compression; the second option is ASTM A48 according to VIKOR and MOORA.

Keywords: brake disc, multi-criteria methods, MCDM.

Resumen

La selección de material para un componente automotor es un proceso complejo, porque implica una exploración de los principales criterios de acuerdo con las propiedades exigidas por el componente a diseñar. El presente estudio tiene como objetivo evaluar un material alterno en la fabricación de un disco de freno en vehículos livianos tipo SUV, a partir de los métodos multicriterio; para lo cual se toman en consideración cinco materiales candidatos para la aplicación deseada (Ti_6Al_4V, $Al_{10}Si$ C, AISI 304L, ASTM A 536 y ASTM A48). Los métodos multicriterio (MCDM) empleados son: VIKOR – la optimización multidisciplinar y solución de compromiso; ELECTRE I – eliminación y opciones que reflejan la realidad; COPRAS – evaluación compleja proporcional; ARAS – evaluación de relación de aditivos; MOORA – optimización multiobjetivo basado en el análisis de radios y el método ENTROPÍA que se emplea para la ponderación de los criterios. Se obtiene que la mejor alternativa es el material ASTM A536 según los métodos COPRAS, ELECTRE I, y ARAS por su baja densidad, un alto límite elástico y una buena resistencia a la compresión; en segunda opción es el ASTM A48 según VIKOR y MOORA.

Palabras clave: disco de freno, métodos multicriterio, MCDM.

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1. Introduction

In the development of the automotive industry, brakes are one of the main safety devices, therefore, the materials to be selected must have the appropriate physical and mechanical properties for optimum performance of the brake disc.

The formation of thermal cracks in the materials used in brake discs depends on thermal fatigue or very severe thermal stresses, produced by the variation of temperature during braking and environmental operating conditions [1]. During braking, the kinetic and potential energy is converted into thermal energy, therefore, it is necessary to know the temperature and thermal stress in braking [2]. It is necessary to investigate the use of new materials that improve braking efficiency and provide greater stability and safety to the vehicle [3]. It is important to select a lightweight alternative material to cast iron which to reduce fuel consumption, depending on its specific weight [4].

The Hierarchical Analytical Process method is used for the environmental evaluation in the selection of composite materials for automotive components, because the available data are difficult to quantify and the characteristics to be evaluated are intangible in an analytical model [5] A systematic and efficient approach to the selection of materials is necessary in order to select the best alternative for a specific engineering application [6]. Multicriteria methods such as COPRAS (Complex Proportional Assessment), VIKOR (from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje,: Multicriteria Optimization and Compromise Solution), ELECTRE I (Elimination and Choice Expressing the Reality), ARAS (additive ratio assessment), MOORA (multi-objective optimization on the basis of ratio analysis) and ENTROPIA which is used to calculate the weight of each criterion, have proven to be adequate methods to validate the selection of materials [5, 6].

In the last 3 years the demand in Ecuador for SUVs of the Chevrolet brand has increased by 7%, with the Suzuki Grand Vitara Sz 2.0 being the fifth most sold vehicle in the country, according to the Association of Automotive Companies of Ecuador [7]. Taking into account that Ecuador is encouraging the inclusion of national products, it is important to select an existing material in the country for the manufacture of the brake disc along with the cost/benefit analysis. The objective of this study is to evaluate an alternative material in the manufacture of a brake disc in light SUV type vehicles, through the COPRAS, VIKOR, ELECTRE I, ARAS, MOORA and ENTROPIA multicriteria methods.

2. Materials and methods

2.1. Definition of the problem

Different types of alloys for the design and manufacture of brake discs in the automotive industry have been developed, because they must meet extremely high parameters, as this device works at high degrees of wear and temperature.

Gray cast iron discs have better wear resistance than alloy or Ti compounds, however, the addition of hard particles to a Ti based compound can substantially improve wear resistance [8]. The analysis of the mechanical properties between an aluminum alloy, cast iron, titanium alloy, ceramic materials and compounds resulted in the most appropriate material for the manufacture of a brake disc to an aluminum alloy [9]. An alternative to metals are composite materials such as high-strength fiber glass, which has greater wear resistance and lighter weight [10].

Thermal conductivity is among the important properties that the selected material must have. A high value allows heat to be dissipated quickly and a high thermal expansion coefficient allows a good thermal expansion when exposing the brake disc to a temperature variation.

In addition a good elastic limit, Young's modulus and a Poisson's coefficient will allow to support high tensions without suffering permanent deformations in the disk. A high value of resistance to compression, traction and Brinell hardness, will prevent the material from fracturing due to the forces produced by the jaws at the time of braking. To reduce the consumption of the vehicle it is necessary to reduce the weight of the vehicle, for this reason the brake disc must have a low density. It is important to carry out a cost-benefit analysis of the selected material.

Taking all these criteria into account, the candidate materials for the manufacture of brake discs in Ecuador are the following: Ti_6Al_4V (titanium alloy, number 1), $Al_{10}Si$ C (aluminum alloy or Duralcan, number 2), AISI 304L (stainless steel, number 3), ASTM A536 (nodular gray cast iron, number 4) and ASTM A48 (pearl gray cast iron, number 5).

2.2. Multi-criteria methods. Pondering criteria

The multicriteria methods used are COPRAS, VIKOR, ELECTRE I, ARAS and MOORA. The calculation of the weights of each criterion is done through the Entropy method, in order to have objective results since it assumes that a criterion has greater weight when there is greater diversity in the evaluation of each alternative.

2.2.1. Entropy method

Entropy measures the uncertainty in the information formulated using the theory of probability. It indicates that a broad distribution represents more uncertainty than a distribution with pronounced peaks. The Entropy method is calculated in the following steps [11]:

Step 1: Construction of the decision matrix.

$$r = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

Step 2: Calculation of the normalized decision matrix P_{ij}, the objective of normalization is to obtain values without dimensions of different criteria to make comparisons between them [11]. It is calculated using equation (1).

$$P_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{1}$$

Step 3: Calculation of entropy E_j, by means of equation (2)

$$E_j = -k \left(\sum_{i=1}^{m} p_{ij} ln(p_{ij}) \right) \tag{2}$$
$$t = 1, 2, 3, \ldots, n.$$

Where $k = \frac{1}{ln\ m}$ it is a constant that guarantees $0 \leq Ej \leq 1$ and m is the number of alternatives.

Step 4: Calculation of criterion diversity D_j, equation (3) allows this parameter to be calculated.

$$D_j = 1 - E_j \tag{3}$$

Step 5: Calculation of the normalized weight of each criterion W_j, by means of equation (4).

$$W_j = \frac{D_j}{\sum_{i=1}^{m} D_j} \tag{4}$$

2.2.2. COPRAS method

The COPRAS method selects the best decision alternatives considering the ideal and worst-ideal solutions, in a classification and step-by-step evaluation of the alternatives in terms of their importance and degree of utility. The algorithm of the COPRAS method consists of the following steps [12]:

Step 1: Calculation of the normalized decision matrix x_{ij}^*, through equation (5).

$$x_{ij}^* = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{5}$$

Step 2: Determine the weighted normalized decision matrix D_{ij}, according to equation (6).

$$D_{ij} = x_{ij}^* \cdot w_j = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix} \tag{6}$$

Where x_{ij}^* is the normalized performance value of i_{th} alternatives in j_{th} criteria and w_j is the weight associated to the j_{th} criteria.

Step 3: The sums S_{i+} and S_{i-} of the weighted normalized values are calculated for both the beneficial and non-beneficial criteria, respectively. These sums S_{i+} and S_{i-} are calculated by means of equations (7) and (8) respectively.

$$Si_{i+} = \sum_{k=1}^{k} D_{ij} \tag{7}$$

$$Si_{i-} = \sum_{k=1}^{k} D_{ij} \tag{8}$$

Step 4: Determine the relative importance of the alternatives Q_i through equation (9).

$$Q_i = S_i + \frac{\sum_{j=1}^{m} S_{i-}}{S_{i-} \sum_{j=1}^{m} \frac{1}{S_{i-}}} \tag{9}$$

The relative importance Q_i of an alternative shows the degree of satisfaction achieved by this alternative.

Step 5: Calculation of the yield index Pi of each alternative, using the following equation (10).

$$P_1 = \frac{Q_i}{Q_{max}} \times 100 \tag{10}$$

Where Q_{max} is the maximum value of relative importance. The value of the performance index P_i is used to obtain a complete classification of the candidate alternatives.

2.2.3. VIKOR method

The basic concept of VIKOR is to first define the ideal positive and negative solutions. The positive ideal solution indicates the alternative with the highest value (score of 100) while the ideal negative solution indicates the alternative with the lowest value (score of 0). The VIKOR commitment algorithm has the following steps [13]:

Step 1: Define the initial decision matrix X_{ij}.

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

Step 2: Calculation of the normalized initial decision matrix f_{ij}, using equation (11).

$$f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{11}$$

Step 3: Determine the best f_i^* and the worst f_i^- value of all the criteria functions of each alternative. By means of equations (12) and (13) respectively.

$$\begin{aligned} f_i^* &= max_j \ f_{ij} \\ i &= 1, 2, 3, \ldots, m \end{aligned} \tag{12}$$

$$\begin{aligned} f_i^- &= min_j \ f_{ij} \\ i &= 1, 2, 3, \ldots, m \end{aligned} \tag{13}$$

Step 4: Calculation of the distance from each value to the positive ideal solution S_i and the distance from each value to the ideal negative solution R_i, by means of equation (14) and (15) respectively.

$$S_i = \sum_{j}^{n} W_i \frac{f_i^* - f_{ij}}{f_i^* - f_i^-} \tag{14}$$

$$R_i = Max_j \frac{W_i f_i^* - f_{ij}}{f_i^* - f_i^-} \tag{15}$$

Step 5: Calculation of the values I_i, para $i = 1, \ldots, I$ is defined by equation (16).

$$I_i = v\left[\frac{s_i - s^*}{s^- - s^*}\right] + (1 - v)\left[\frac{R_i - R^*}{R^- - R^*}\right] \tag{16}$$

Where $S^* = Min\ S_i$, $S^- = Max\ S_i$, $R^* = Min\ R_i$, $R^- = Max\ R_i$, and v is a weighting reference ($v > 0.5$). $\frac{(R_i - R^*)}{(R^- - R^*)}$, represents the distance of the ideal negative solution of i_{th} values.

Step 6: The ranking is determined, the highest value is the best alternative

2.2.4. ELECTRE I Method

The ELECTRE I method has the ability to handle discrete quantitative and qualitative criteria and provides a complete order of alternatives. The limitation is replaced by the concordance and discordance of the matrix index. The procedure of the ELECTRE I method is as follows [14]:

Step 1: Define the initial decision matrix r_{ij}.

$$r_{ij} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

Step 2: Normalization of the decision matrix, this process will allow transforming different scales and units among several common criteria that allow comparisons accross criteria, according to equation (17).

$$R_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} {r_{ij}}^2}} \tag{17}$$

PStep 3: Construction of the normalized weighted decision matrix V_{ij}. For which the normalized decision matrix R_{ij} is multiplied with its respective weight, expressed in equation (18).

$$V_{ij} = W_i \times R_{ij} \tag{18}$$

$$V_{ij} = \begin{bmatrix} W_1 r_{11} & W_2 r_{12} & \cdots & W_n r_{1n} \\ W_2 r_{21} & W_2 r_{22} & \cdots & W_n r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ W_n r_{m1} & W_n r_{m2} & \cdots & W_n r_{mn} \end{bmatrix}$$

Step 4: Calculation of the intervals of agreement (C_{ab}) and disagreement (D_{ab}), that is, C_{ab} indicates the most preferable alternative and D_{ab} indicates the least preferable alternative. Equations (19) and (20) are used respectively.

$$C_{ab} = \{j | x_{aj} \geq x_{bj}\} \tag{19}$$

$$D_{ab} = \{j | x_{aj} \leq x_{bj}\} = j - C_{ab} \tag{20}$$

Step 5: Determination of the agreement interval matrix C_{ab}, which is obtained by adding the weights to the weights associated with the criteria in which the alternative i is better than the alternative j or vice versa; in case of a tie, half of the weight is assigned to each of the alternatives according to equation (21).

$$C_{ab} = \sum_{j=C_{ab}} W_j \tag{21}$$

Step 6: Determination of the discordance index matrix D_{ab}, which is calculated as the largest difference between the criteria for which the alternatives i is dominated by the j, then dividing by the greater difference in absolute value between the results obtained by the alternative i and j, according to equation (22).

$$D_{ab} = \frac{\left(\substack{max \\ j \in D_{ab}}\right) |V_{aj} - V_{bj}|}{\left(\substack{max \\ j \in J, m, n \in I}\right) |V_{mj} - V_{nj}|} \tag{22}$$

Step 7: Calculation of the maximum threshold $\bar{c}$ for the concordance index and the maximum threshold $\bar{d}$ for the discordance index, by means of equations (23) and (24) respectively.

$$\bar{c} = \sum_{a=1}^{m} \sum_{b}^{m} \frac{c(a, b)}{m(m-1)} \tag{23}$$

$$\bar{d} = \sum_{a=1}^{m} \sum_{b}^{m} \frac{c(a,b)}{m(m-1)} \tag{24}$$

Step 8: Calculation of the dominant concordance matrix. Once the concordance indexes and the minimum agreement threshold have been determined, the dominant agreement matrix is calculated with the following condition:

$$cd_{ij} \begin{cases} e(a,b) = 1 & \text{si } c(a,b) \geq \bar{c} \\ e(a,b) = 0 & \text{si } c(a,b) < \bar{c} \end{cases}$$

Step 9: Calculation of the dominant discordant matrix. In the same way as the previous one, the values of the matrix of dominant discordance are obtained from the matrix of discordance index and the maximum threshold of discordance d̄. By the following condition.

$$dd_{ij} \begin{cases} f(a,b) = 1 & \text{si } d(a,b) \geq \bar{d} \\ f(a,b) = 0 & \text{si } d(a,b) < \bar{d} \end{cases}$$

Step 10: Calculation of the upper and lower net value C_a and D_a, by means of equations (25) and (26) respectively.

$$C_a = \sum_{b=1}^{n} c_{(a,b)} - \sum_{b=1}^{n} c_{(b,a)} \tag{25}$$

$$D_a = \sum_{b=1}^{n} d_{(a,b)} - \sum_{b=1}^{n} d_{(b,a)} \tag{26}$$

Where C_a is the sum of the competitive superiority number of all the alternatives and D_a is used to determine the inferiority number by classifying the alternatives.

2.2.5. ARAS method

The ARAS method determines the complex relative efficiency of a feasible alternative that is directly proportional to the relative effect of the values and weights of the main criteria considered. Based on the theory of utility and the quantitative method. The steps of this method are the following [15].

Step 1: Conformation of the decision matrix X_{ij},

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

Step 2: Calculation of the normalized decision matrix ($\overline{X_{ij}}$), taking into account the beneficial values calculated with equation (27).

$$\overline{X_{ij}} = \frac{x_{ij}}{\sum_{i=0}^{m} X_{ij}} \tag{27}$$

$$\bar{X}_{ij} = \begin{bmatrix} \bar{x}_{11} & \bar{x}_{12} & \cdots & \bar{x}_{1n} \\ \bar{x}_{21} & \bar{x}_{22} & \cdots & \bar{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{x}_{m1} & \bar{x}_{m2} & \cdots & \bar{x}_{mn} \end{bmatrix}$$

The non-beneficial criteria are calculated by means of equation (28).

$$X_{ij} = \frac{1}{X_{ij}^{*}}; \overline{X_{ij}} = \frac{x_{ij}}{\sum_{i=0}^{X_{ij}}} \tag{28}$$

Step 3: Calculation of the weighted normalized decision matrix is done with equation (29).

$$\hat{X}_{ij} = \bar{X}_{ij} \times W_j \tag{29}$$

$$\hat{X}_{ij} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{x}_{m1} & \hat{x}_{m2} & \cdots & \hat{x}_{mn} \end{bmatrix}$$

The weight values W_j are determined by the Entropy method.

Where W_j is the criterion weight j and $\bar{X}_{ij}$ it is the standardized classification of each criterion.

Step 4: Calculation of the optimization function S_i using equation (30).

$$S_i = \sum_{j=1}^{n} \hat{X}_{ij} \tag{30}$$

Where S_i is the value of the optimization function of the alternative i. This calculation has a directly proportional relationship with the process of the values X_{ij} and weights W_j of the criteria investigated and their relative influence on the final result.

Step 5: Calculation of the degree of utility. This degree is determined by comparing the variant that is under analysis with the best S_o, according to equation (31).

$$K_i = \frac{s_i}{s_o} \tag{31}$$

Where S_i and So are the values of the optimization function. These values range from 0% to 100%, therefore the alternative with the highest K_i is the best of the alternatives analyzed.

2.2.6. MOORA method

The MOORA method begins from reference points. These references will be the highest evaluation of the radius vector of alternatives with respect to each criterion, whether maximum or minimum. The steps of this method are described as follows [16].

Step 1: Determination of the initial decision matrix X_{ij}.

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

Step 2: Calculation of the radius matrix of the form $X_{ij} = [(\overline{x_{ij}})]$ to normalize the initial decision matrix, equation (32) is used.

$$\overline{X_{ij}} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{32}$$

Step 3: The weight vector of the criteria is defined.

$$W = |W_1 \;\; W_2 \;\; W_3 \;\; \ldots \;\; W_n|$$

Step 4: Calculation of the matrix normalized by weights. It is weighted by multiplying the standardized deduction matrix by the weights of each criterion.

Step 5: The aggregation function is determined to evaluate each alternative $S(x_i)$, using equation (33).

$$S(x_i) = \sum_{i=1}^{h} \overline{X_{ij}} - \sum_{i=h+1}^{h} \overline{X_{ij}} \tag{33}$$

Where $i = 1, 2, 3, \ldots, h$ corresponds to the criteria cataloged as a maximum; $i = h + 1, h + 2, \ldots n$ corresponds to the criteria cataloged as a minimum.

Step 6: The preference ranking is determined. The best alternative is the one with the highest $S(x_i)$ value.

3. Results and discussion

3.1. Application of the entropy method

The candidate materials and the criteria under analysis are shown in Table 1. The properties of the alternatives are: density (A), price (B), Young's modulus (C), elastic limit (D), Poisson's radius (E), tensile strength (F), compressive strength (G), Brinell hardness (H), thermal conductivity (I) and coefficient of thermal expansion (J). The Entropy method is applied for the weighting criteria, in order to obtain objective weights at the time of the evaluation, since it is based on defined mathematical models; unlike the AHP method that is based on expert criteria applied by [14].

Table 2 shows the normalized decision matrix of the Entropy method, which is calculated according to equation (1). The values of the entropy E_j of each variable, the diversity of criteria (D_j) and the normalized weights of each criterion (W_j) are indicated in Table 3, according to equations (2), (3) and (4) respectively.

Table 1. Evaluation matrix

Material	Density (kg/m^3) A	Price (USD/kg) B	Young's modulus (GPa) C	Elastic limit (MPa) D	Poisson's radius Poisson E	Tensile strength (MPa) F	Compression resistance (MPa) G	Brinell hardness (HV) H	Thermal conductivity (W/m°C) I	Coefficient of thermal expansion (µstrain/°C) J
Ti_6Al_4V	4430	27.5	115	898	0.349	620	848	347	8.91	9.1
$Al_{10}SiC$	2770	8.29	88	358	0.32	372	358	118	148	18
AISI 304L	7980	4.53	205	310	0.275	620	310	210	16	18
ASTM A536	7150	0.67	173	339	0.28	500	351	217	41	12.5
ASTM A48	7200	0.67	120	149	0.265	250	170	252	46	13

Table 2. Normalized decision matrix P_{ij}

A	B	C	D	E	F	G	H	I	J
0.150	0.660	0.164	0.437	0.234	0.262	0.416	0.303	0.034	0.128
0.093	0.199	0.125	0.174	0.214	0.157	0.175	0.103	0.569	0.255
0.270	0.108	0.292	0.150	0.184	0.262	0.152	0.183	0.061	0.255
0.242	0.016	0.246	0.165	0.188	0.211	0.172	0.189	0.157	0.177
0.243	0.016	0.171	0.072	0.178	0.105	0.083	0.220	0.177	0.184

Table 3. Calculation E_i, D_j and W_j according to the Entropy method

Criteria	E_i	D_j	W_j
A	0.961	0.038	0.038
B	0.602	0.397	0.399
C	0.971	0.028	0.028
D	0.894	0.105	0.106
E	0.996	0.003	0.003
F	0.969	0.030	0.031
G	0.911	0.088	0.088
H	0.966	0.033	0.033
I	0.749	0.250	0.251
J	0.981	0.018	0.019

3.2. COPRAS

The normalized decision matrix (x^*_{ij}), is calculated with equation (5), while the normalized matrix by weight (D_{ij}) is calculated according to equation (6) represented in Table 4. The sum of the weighted normalized values (S_{i+}), (S_{i-}) the relative importance (Q_i) shows the degree of satisfaction of an alternative and the performance index (P_i) that determines the ranking of candidate materials for the manufacture of a brake disc, are calculated with equations (7), (8), (9) and (10) respectively and all these calculations are indicated in Table 5, where the best material is 4 (ASTM A536) due to the selection of the best decision alternatives related to Young's modulus (C), elastic limit (D), Poisson radius (E), tensile-compression resistance (F and G), hardness (H) and thermal conductivity (I).

Table 4. Standard decision matrix of weights D_{ij} of the COPRAS method

Material	A	B	C	D	E	F	G	H	I	J
1	0.005	0.263	0.004	0.046	0.0008	0.008	0.037	0.010	0.008	0.002
2	0.003	0.079	0.003	0.018	0.0007	0.004	0.015	0.003	0.143	0.004
3	0.010	0.043	0.008	0.016	0.0006	0.008	0.013	0.006	0.015	0.004
4	0.009	0.006	0.007	0.017	0.0007	0.006	0.015	0.006	0.039	0.003
5	0.009	0.006	0.004	0.007	0.0006	0.003	0.007	0.007	0.044	0.003

Table 5. Calculation S_{i+}, S_{i-}, Q_i, P_i and COPRAS Ranking

Material	S_{i+}	S_{i-}	Q_i	P_i	Ranking
1	0.118	0.269	0.128	47.68	4
2	0.195	0.083	0.227	84.65	3
3	0.073	0.053	0.123	45.93	5
4	0.096	0.015	0.269	100.0	1
5	0.079	0.015	0.251	93.35	2

3.3. VIKOR

The normalized initial decision matrix f_{ij} is presented in Table 6, these values are obtained by means of equation (11). The best and worst value is determined with equations (12) and (13) respectively, which is shown in Table 7. The values of the distance from each value to the positive solution (S_i), is calculated according to equation (14), is indicated in Table 8 and the distance to the ideal negative solution (R_i), is calculated with the equation (15), which is shown in Table 9. The value of (I_i) is obtained by equation (16), the highest value of (I_i) determines the best material in this case is an ASTM A48 (number 5). These values are indicated in Table 10, due to their low density (A), low Poisson radius (E) and high Brinell hardness (H).

Table 6. Normalized decision matrix F_{ij} with the VIKOR method

Material	A	B	C	D	E	F	G	H	I	J
1	0.318	0.945	0.351	0.831	0.521	0.561	0.810	0.645	0.055	0.280
2	0.198	0.284	0.268	0.331	0.477	0.336	0.342	0.219	0.917	0.553
3	0.573	0.155	0.625	0.286	0.410	0.561	0.296	0.390	0.099	0.553
4	0.513	0.023	0.528	0.313	0.418	0.452	0.335	0.403	0.254	0.384
5	0.517	0.023	0.366	0.137	0.395	0.226	0.162	0.468	0.285	0.400

Table 7. Ideal and non-ideal solution according to VIKOR

	A	B	C	D	E	F	G	H	I	J
fi*	0.573	0.945	0.625	0.831	0.521	0.561	0.810	0.645	0.917	0.553
fi-	0.198	0.023	0.268	0.137	0.395	0.0226	0.162	0.219	0.055	0.280

Table 8. Calculations S_i, S_{imax} and S_{imin}

Material	S_i	S_{imax}	S_{imin}
1	0.319		
2	0.548		
3	0.757	0.864	0.319
4	0.795		
5	0.864		

Table 9. Calculations R_i, R_{imax} and R_{imin}

Material	R_{i+}	R_{imax}	R_{i-}	R_{imin}
1	0.251		0.000	
2	0.285		0.000	
3	0.341	0.399	0.000	0.000
4	0.399		0.002	
5	0.399		0.003	

Table 10. Calculations of I_i for v =0.5 and VIKOR Ranking

Material	Ii	***Ranking***
1	0.315	5
2	0.568	4
3	0.830	3
4	0.936	2
5	1.000	1

3.4. ELECTRE I

The data of the initial decision matrix is tabulated in Table 1 and the weighted standard decision matrix (V_{ij}) is obtained using equation (18), said values are indicated in Table 11. The matrix of concordance intervals (C_{ab}), is calculated according to equation (19) and is shown in Table 12. By means of equation (20) the matrix values of discordance intervals (D_{ab}) are calculated, which are tabulated in Table 13. The maximum threshold ($\bar{c}$) for the concordance index, is determined with equation (23) and the dominant concordance matrix (cd_{ij}) is represented in Table 14. While the maximum threshold for the discordance index ($\bar{\mathrm{d}}$), is calculated according to equation (24), tabulated in Table 15 and the jarring matrix (dd_{ij}) is shown in Table 16. Finally, the upper and lower net value (C_a) and (C_b), is obtained according to equations (25) and (26) respectively, these values are indicated in Table 17. The material with the best score is ASTM A536. The materials with the best score are the $Al_{10}SiC$ (number 2) and the ASTM A536 (number 4), with thermal conductivity (I), elastic limit (D) and tensile-compression resistance (F and G) as determining factors.

Table 11. Weighted normalized decision matrix V_{ij} according to ELECTRE I

Material	A	B	C	D	E	F	G	H	I	J
1	0.026	0.021	0.009	0.088	0.001	0.017	0.071	0.021	0.013	0.005
2	0.030	0.285	0.007	0.035	0.001	0.010	0.030	0.007	0.231	0.010
3	0.016	0.337	0.017	0.030	0.001	0.017	0.026	0.013	0.025	0.010
4	0.018	0.390	0.015	0.033	0.001	0.014	0.029	0.013	0.064	0.007
5	0.018	0.390	0.010	0.014	0.001	0.007	0.014	0.015	0.071	0.007

Table 12. Interval concordance matrix C_{ab}

	Alt. 1	Alt. 2	Alt. 3	Alt. 4	Alt. 5
Alt. 1	0.000	0.291	0.285	0.301	0.301
Alt. 2	0.708	0.000	0.498	0.507	0.539
Alt. 3	0.714	0.501	0.000	0.078	0.275
Alt. 4	0.698	0.492	0.921	0.000	0.496
Alt. 5	0.698	0.461	0.724	0.504	0.000

Table 13. Array of discrepancy intervals D_{ab}

	Alt. 1	Alt. 2	Alt. 3	Alt. 4	Alt. 5
Alt. 1	0.000	0.201	0.183	0.149	0.199
Alt. 2	1.000	0.000	1.000	1.000	1.000
Alt. 3	1.000	0.250	0.000	0.063	0.337
Alt. 4	1.000	0.626	1.000	0.000	1.000
Alt. 5	1.000	0.656	1.000	0.418	0.000

Table 14. Dominant concordance matrix cd_{ij} and concordance threshold $\bar{\mathrm{c}}$

	Alt. 1	Alt. 2	Alt. 3	Alt. 4	Alt. 5	$\bar{\mathrm{c}}$
Alt. 1	0.000	0.000	0.000	0.000	0.000	
Alt. 2	1.000	0.000	0.000	1.000	1.000	
Alt. 3	1.000	1.000	0.000	0.000	0.000	0.5
Alt. 4	1.000	0.000	1.000	0.000	0.000	
Alt. 5	1.000	0.000	1.000	1.000	0.000	

Table 15. Dominant disagreement matrix dd_{ij} and discordance threshold $\bar{\mathrm{d}}$

	Alt. 1	Alt. 2	Alt. 3	Alt.4	Alt. 5	$\bar{\mathrm{d}}$
Alt. 1	1.000	1.000	1.000	1.000	1.000	
Alt. 2	0.000	1.000	0.000	0.000	0.000	
Alt. 3	0.000	1.000	1.000	1.000	1.000	0.654
Alt. 4	0.000	1.000	0.000	1.000	0.000	
Alt. 5	0.000	0.000	0.000	1.000	1.000	

Table 16. Matrix of aggregate dominance (concordance-discordant) acd_{ij}

	Alt. 1	**Alt. 2**	**Alt. 3**	**Alt.4**	**Alt. 5**
Alt. 1	0.000	0.000	0.000	0.000	0.000
Alt. 2	0.000	0.000	0.000	0.000	0.000
Alt. 3	0.000	1.000	0.000	0.000	0.000
Alt. 4	0.000	0.000	0.000	0.000	0.000
Alt. 5	0.000	0.000	0.000	1.000	0.000

Table 17. Calculation of the upper and lower net value D_{ai} and ELECTRE I Ranking

Materiales	C_{ai}	D_{ai}	***Ranking***
1	0.0000	0.000	2
2	10.000	–1.000	1
3	–1.0000	1.000	3
4	10.000	–1.000	1
5	–1.0000	1.000	3

3.5. ARAS

According to equation (27) the normalized decision matrix is calculated ($\bar{\mathrm{X}}_{\mathrm{ij}}$), taking into account the calculation of the non-beneficial values by means of equation (28). Subsequently, the decision matrix normalized by weight ($\hat{\mathrm{X}}_{\mathrm{ij}}$) is defined by equation (29), whose values are presented in Table 18. Using equation (30) to calculate the values of the optimization function (S_i) of each of the alternatives, the degree of utility (K_i) is calculated by means of equation (31), which determines the ranking of the alternatives for the application under study. These values are shown in Table 19, showing that the material ASTM A536 (number 4) is the best as a result of the relative effect of the values of thermal conductivity, yield strength and compressive strength.

Table 18. Weighted normalized decision matrix $\hat{X}_{ij}$, of the ARAS method

Material	**A***	**B***	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
1	0.008	0.004	0.004	0.046	0.0008	0.008	0.037	0.010	0.008	0.002
2	0.014	0.014	0.003	0.018	0.0007	0.004	0.015	0.003	0.143	0.004
3	0.004	0.026	0.008	0.016	0.0006	0.008	0.013	0.006	0.015	0.004
4	0.005	0.177	0.007	0.017	0.0007	0.006	0.015	0.006	0.039	0.003
5	0.005	0.177	0.004	0.007	0.0006	0.003	0.007	0.007	0.044	0.003

Table 19. Calculations S_i, K_i and Ranking

Material	S_i	K_i	***Ranking***
1	0.131	0.470	4
2	0.223	0.800	3
3	0.104	0.373	5
4	0.279	1.000	1
5	0.261	0.9383	2

3.6. MOORA

The decision matrix $(\overline{X_{ij}})$ is obtained according to equation (32). Table 20 shows the weighted normalized decision matrix. Then we obtain the aggregation function S(xi) that evaluates each alternative by means of equation (33), and this calculation also determines the preference ranking of each alternative. The values are shown in Table 21, showing that the material Al_{10} Si C (number 2) is the best because its thermal conductivity (I) and coefficient of thermal expansion (J) are high compared to the rest of the materials experienced.

Table 20. Weighted normalized decision matrix $\hat{X}_{ij}$, by the MOORA method

Material	A	B	C	D	E	F	G	H	I	J
1	0.012	0.377	0.009	0.088	0.001	0.017	0.071	0.021	0.013	0.005
2	0.007	0.113	0.007	0.035	0.001	0.010	0.030	0.007	0.231	0.010
3	0.022	0.062	0.017	0.030	0.001	0.017	0.026	0.013	0.025	0.010
4	0.019	0.009	0.015	0.033	0.001	0.014	0.029	0.013	0.064	0.007
5	0.019	0.009	0.010	0.014	0.001	0.007	0.014	0.015	0.071	0.007

Table 21. Aggregation function S(xi) and Ranking MOORA

Material	S(xi)	*Ranking*
1	–0.159	5
2	0.212	1
3	0.057	4
4	0.149	2
5	0.113	3

3.7. EVALUATION OF THE MCDM

The MCDM has the task of classifying a finite number of decision alternatives, each of which is explicitly described in terms of different decision criteria that must be taken into account simultaneously. For this reason, these methods are used in the selection of the material for the construction of a brake disc.

Figure 1 shows the ranking of all MCDM methods, with the observation that the COPRAS and ARAS method have the same ranking values, so their curves overlap.

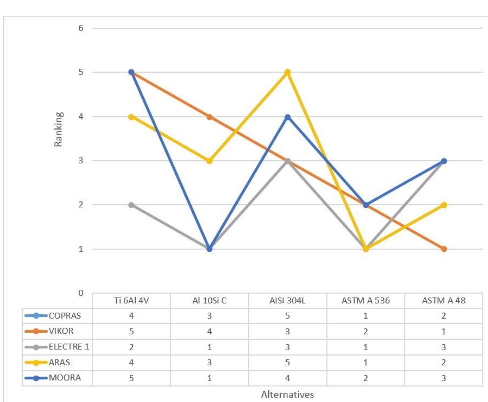

Figure 1. Ranking of the alternatives according to the MCDM methods

The best material in the COPRAS, ELECTRE I, and ARAS methods is ASTM A536, because of its low density (A), high elastic limit (D) and good compressive strength (G), the MOORA and VIKOR method place it as a second alternative. The second best option evaluated is the Al 10Si C and the ASTM A48 by the criteria of ELECTRE I, MOORA and VIKOR, since it has good thermal conductivity (I), low density (A) and an accessible price (B). These results are aligned with the materials used in the study conducted by Maleque, Dyuti, & Rahman [9]. In addition, Kharate & Chaudhari [17] study the effect of material properties on the noise and performance of the brake disc by the FEM and EMA approach, for which they experiment with gray cast iron, ceramic coal and steel, obtaining as a result that the gray cast iron has a natural frequency lower than the rest of the materials tested.

4. Conclusions

The MCDM methods used in this investigation allowed the selection of a material for the manufacture of a brake disc, incorporating quantitative and qualitative criteria. The weighting of the properties of the candidate materials for the construction of a brake disc was obtained by the ENTHROPY method. According to the COPRAS, ELECTRE I and ARAS methods, the best material is ASTM A536, with better thermal and mechanical properties. A second option according to the criteria of ELECTRE I, MOORA and VIKOR are the $Al_{10}Si$ C and the ASTM A48. The MCDM techniques allow solving complex problems, which adapt to any type of need and apply to different areas of engineering.

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