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Mathematical representations of 5 and 6-year-old children when solving an open-ended problem

Representaciones matemáticas de niños y niñas de 5-6 años cuando resuelven un problema abierto

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Abstract

Problem solving and representation are two fundamental processes of mathematical activity. Their development provides a key basis for learning mathematics at all school levels. Hence the importance of promoting these processes from an early age. The aim of this article is to describe the representations and ways of solution posed by a group of children in pre-school education (5-6 years), in a Catalan school, when solving an open-ended arithmetic problem. The study follows a descriptive-interpretative methodology. A school task is designed and implemented from which individual written productions are obtained. In addition, interviews were conducted with each of the students and the corresponding video recordings were made. The data are systematised and a two-phase analysis is carried out: initially the types of representation are characterised and then the calculation methods used by the children. The results indicate that all the participating pupils produce representations to solve the problem. All the children make iconic representations, and a few combine iconic and symbolic representations. As for the ways of solving the problem, continuous counting predominates, although in some cases proposals are made in which more complex reasoning is evident. In these cases, the children propose groupings which are expressed by means of drawings and symbols.

Keywords: Problem solving, representation, reasoning, calculation methods, mathematical activity, early ages.

Resumen

La resolución de problemas y la representación son dos procesos fundamentales de la actividad matemática. Su desarrollo proporciona una base clave para el aprendizaje de las matemáticas en todos los niveles escolares. Por ello, la importancia de la promoción de estos procesos desde las primeras edades. El objetivo de este artículo es describir las representaciones y formas de solución planteadas por un grupo de 23 niñas y niños de educación infantil (5-6 años), de una escuela catalana, cuando resuelven un problema aritmético abierto. El estudio sigue una metodología descriptiva-interpretativa. Se diseña e implementa una tarea escolar de la que se obtienen producciones escritas individuales. Se realizan además entrevistas a cada uno de los alumnos y se cuenta con los registros en video correspondientes. Los datos se sistematizan y se realiza un análisis en dos fases: inicialmente se caracterizan los tipos de representación y luego los métodos de cálculo planteados por los niños. Los resultados indican que los alumnos participantes elaboran representaciones para resolver el problema. Todos los niños y las niñas realizan representaciones icónicas, y algunos pocos combinan representaciones icónicas y simbólicas. En cuanto a las formas de solución del problema predomina el conteo continuo, aunque en algunos casos se realizan propuestas en las que se evidencian razonamientos más complejos. En estos casos, los niños[1] plantean agrupaciones las cuales se expresan mediante dibujos y símbolos.

Descriptores: Resolución de problemas, representación, razonamiento, métodos de cálculo, actividad matemática, edades iniciales.

1 This article uses inclusive terms such as el maestro, el docente, el estudiante, el niño and el profesor, and their corresponding plurals (as well as other words in the educational context) to refer to men and women. This option is because there is no universal agreement on how to refer jointly to both sexes in the Spanish language, except by using o/a, los/las and other similar words, and this type of formula implies a graphic saturation that can make it difficult to understand the reading of the text.

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1 Introduction and state-of-the-art

Current academic approaches to early childhood education mention the importance of globalized approaches, interdisciplinarity and the need for competency development (National Council of Teachers of Mathematics-NCTM, 2000; 2014; Ministerio de Educación y Formación Profesional, 2022; National Association for the Education of Young Children-NAEYC, 2020). Authors such as Clements and Sarama (2016); De Castro *et al.* (2012); Vanegas and Giménez (2018), among others, highlight the role of mathematical processes in the acquisition of competencies and point out that these are essential to promoting the ability to use mathematics in a comprehensive and effective way in different contexts. Supporting and enriching these processes and promoting the development of children's mathematical thinking is one of the challenges of early childhood education (Baroody, 2003; Cheeseman, 2019; Clements and Sarama, 2021; Ginsburg and Amit, 2008: Lopes *et al.*, 2017). As a result, raising and solving problems, analyzing different strategies and solutions, and reflecting on them should be main activities in the teaching and learning processes of mathematics at each school level (NAEYC and NCTM, 2013; Mason, 2016; Schoenfeld, 2016).

According to Edo (2005), math learning is a socially mediated construction process. It is especially relevant when thinking about early childhood education, as it involves assuming that children do not learn by receiving and passively accumulating information from the environment, but they do so through an active meaning-making and sense-making process, where problem-solving, communication and representation are essential processes (Battista, 2016). If mathematics is considered as the result of certain actions carried out by people and as a changing phenomenon, mathematical activity must be characterized by the desire to find something: data, processes, relationships, results, a way of communicating, etc. Therefore, early mathematics education should focus on helping children live mathematical activity situations, i.e., search situations where the focus is the practices of children.

As Baroody (1993), Saundry and Nicole (2006) and Carruthers and Worthington (2010) mention, representations and drawing are fundamental tools for solving problems in early ages. These authors argue that representations are essential in the construction of meanings because representations help children to concretize the problems and decide the procedure to use in their solution. Carruthers and Worthington (2009) also stress the importance that teachers recognize representations made by children while solving problems. In this way they will be better identify the ideas and ways of reasoning. For this reason, it is necessary to explore the type of representations and how children in early education solve problems. The purpose of this article is to describe the type of representations and strategies used by a group of children (5-6 years old) when involved in an open problem-solving task.

This study is based on two main aspects. The first refers to the solution of open problems at an early age and the second relates to the use of representations and their importance in solving and communicating problems.

Ramírez and de Castro (2014) say that it is essential to introduce problem solving in early childhood education, since it not only encourages the development of informal strategies but also because it helps children to give meaning to arithmetic operations and certain procedures they will learn as they advance in their schooling. We agree with Alsina (2012) who, following the NCTM (2000) approaches, proposes that there are four aspects concerning the solution of problems that should be worked from the early childhood education: a) to construct mathematical knowledge through problem solving, to propose a variety of contexts; b) solve problems arising from mathematics and different contexts, everyday situations, daily routines, experimentation situations, among others; c) apply and adapt a variety of appropriate strategies to solve problems, such as asking good questions; encourage interaction, negotiation and dialog in the classroom; etc.; and, d) regulate and reflect on the process of solving mathematical problems.

While it is true that there are different types of mathematical problems (realistic, authentic, open, among others), teachers are the ones who choose to use one or the other regarding the objectives proposed. According to Pehkonen (1997), an open problem is one where the starting situation is opened, as opposed to closed problems where the beginning and end are exactly explicit. In this group, Baroody (1988) refers to routine and non-routine verbal statement problems. Non-routine problems are those that involve different procedures for their solution, and may have different answers. Routine or non-routine problems are those of division, which involve the action of separating the total parts into units or as wholes. In the investigation of Saoundry and Nicole (2006) two types of non-routine division problems are presented: A) arithmetic problems arising from a grouping (set of elements), which must then be distributed; and b) arithmetic problems that also start from a grouping (set of elements), but where their division involves more than one operation to solve it (a set of elements must be divided into subsets).

In the early ages, representations serve both to build new mathematical knowledge and to express mathematical ideas (NCTM, 2000). In this sense, Burgués and Sarramona (2013) argue that it is desirable for mathematical language to become a natural form of expression in the classroom among teachers and children. To achieve this objective, the conversation about mathematics must be promoted, first through verbal language, and progressively introduce the terms and forms proper to mathematical language (oral and written). It is not about children memorizing symbols, but learning to communicate their mathematical ideas with meaning, hence the importance to explore their representations.

Teachers must analyze the representations of their students and their discussions (their mathematical communication) to evaluate the development of their mathematical thought and thus offer them the necessary elements to connect their own languages to the conventional mathematical language (NCTM, 2000; Edo *et al.,* 2009). The truth is that children naturally represent cognitive ideas through paper, objects, play, etc., in short, they use the representations to shape their schemes and make them meaningful on paper (Carruthers and Worthington, 2006; Worthington, 2009).

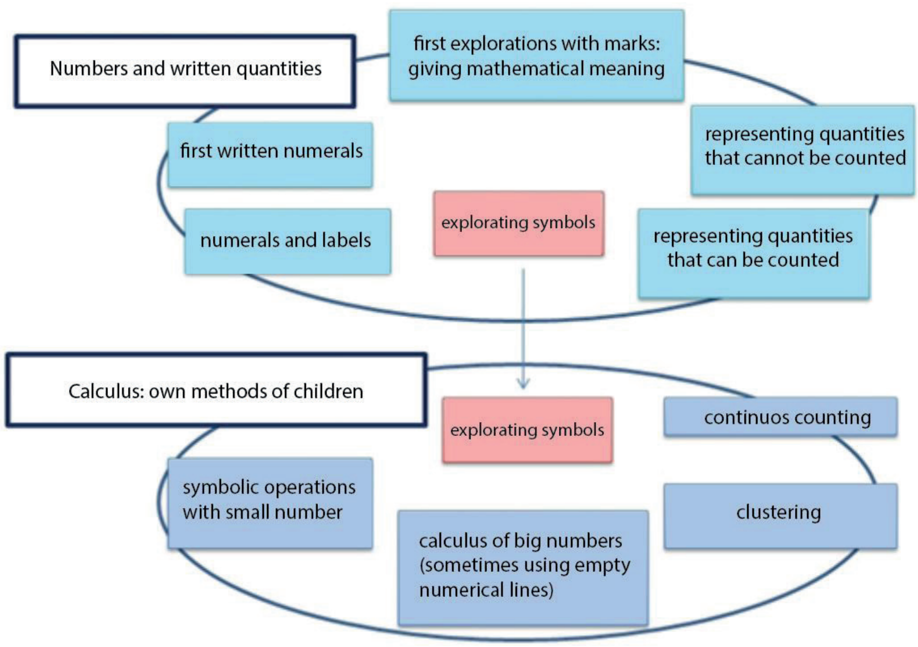
Various authors have studied the mathematical representations of children in the early ages. Thus, Deliyianni *et al.* (2009) studied the ways of representation generated by the students of pre-school and elementary school by examining the compliance with the norms during the didactic process in the solution of problems. While Nicol and Saundry (2006) investigated how children of the early ages think mathematically and represent an arithmetic problem, Smith (2003) and Woleck (2001) argue that drawings perform two fundamental functions: a) they serve to model problems and b) they are the support of mathematical activity that allows them to be solved. In addition, they point to the importance of listening to the students' explanations of their drawings to understand the mathematical activity they perform. In the case of Carruthers and Worthington (2006), from the analysis of mathematical drawings and writings by children up to the age of eight, they identify five types of graphics: dynamic, pictographic, iconic, symbolic, and written. For Carruthers and Worthington (2005, 2006), children reach the mathematical meaning of abstract symbols from their own representations and by constructing their own meaning.

Carruthers and Worthington (2006) propose two dimensions for interpreting the mathematical graphs of boys and girls aged from 0 to 8 years old. The first concentrates on *written repre-*

sentations of numbers and quantities and the second focuses on the *written calculation methods* devised by children. These authors also produce a non-hierarchical taxonomy (Figure 1) illustrating the categories considered in each of the dimensions identified for the characterization of children's mathematical graphs (Carruthers and Worthington, 2013).

Figure 1

Translation Taxonomy: children's mathematical graphs



Note. Carruthers and Worthington (2013).

In the first dimension: Written representations of numbers and quantities, five categories are considered:

- First explorations with 'marks', sometimes these first representations are seen by adults as simple scribbles, but they are an important step for children on their way to multidimensional representations of their world.
- *First written numerals* or when children refer to their marks as numbers. At this point children understand which numbers and letters have meaning and begin to make a general differentiation between them: "this is a number," even though they are not yet recognized as numbers, but they may have numerical qualities.
- *Numerals as labels*, in this case children identify numbers and letters in their surroundings and show interest in using them; they observe the function of written numbers in a social sense. There is a time when children move from identifying these symbols in their surroundings to write them for their own purposes. This is a significant change because by choosing to write these numbers they convert what they read into a standard symbolic language and choose to use them in meaningful contexts.
- *Represented quantities that are not counted*, these are made by younger children, and are live and not very accurate representations. For example, the case of a three-year-

old boy who represents an eight-legged spider, but the representation shows a spider with many legs, more than eight. It is clear that the child represents his or her personal sense of quantity, not a concrete amount.

- *Represented quantities that are counted,* in this case children make representations of amounts and count them, for example, by drawing vertical lines and saying that they are "rain drops" and counting them at the end. For Carruthers and Worthington (2010), uncounted quantities precede counted quantities, but there may be an overlap between these two aspects. In turn, this developmental aspect leads children directly to the beginning of written calculations.

According to Carruthers and Worthington (2006), as children explore calculations in a variety of ways, their own representations support their mental methods and help them calculate. The count has a strong presence at the beginning of the written calculus (Clements and Sarama, 2013; Baroody *et al.*, 2019)

In the second dimension: *Calculation methods* devised by children, the following categories are described:

- *Continuous count* refers to the first representations children make for addition and subtraction. Various studies show that young children make simple additions and subtractions with counting strategies, telling everything, all the elements. So, if there are two sets, they count one and when they finish, they continue with the next, without separating the two sets.
- *Separation of sets*, in this case children exhibit different strategies to show that two amounts are separated. They make groupings of two or more sets of elements that must be added, placing each on one side of the sheet of paper or leaving a space between them; separating the sets with words; placing a vertical line between the sets, among others.
- *Exploration of symbols,* in this case children begin to make explicit use of symbols (invented or sometimes using standard symbols). It is also considered when children make marks in their procedures that show that they understand the symbols, even if they do not appear explicitly.
- *Symbolic operations with small numbers*, at this point children already know the standard symbols and understand their role and have developed strategies to solve problems.
- *Calculations with large numbers* (sometimes using annotations or empty number lines). Calculating with large numbers is more difficult, as it is needed to understand what the large numbers involved look like and may need to manipulate several steps. This is where mental methods and some taught supports can be valuable, such as the number line.

It should be mentioned that children's responses to a certain mathematical task involved in representations can be classified into several of these categories, not necessarily one.

There has been a common questioning about how young children can solve mathematical problems, since most of them do not know how to read or write. This type of question reveals a misconception which must be solved (Lopes *et al.*, 2017). It is important to understand that thinking and language are linked, and that representations play a fundamental role in children's ways of thinking and communication. Investigating how children respond when they have a mathematical problem; the types of drawings they do spontaneously; the things they think while drawing; the relationships they establish and express both orally and in writing is key to understanding how they construct their mathematical ideas.

2 Methodology

In this research a descriptive-interpretative methodology was used (Latorre *et al.,* 2003; Cohen *et al.,* 2018). A descriptive analysis of the data is made, to later relate and interpret these descriptions considering the theoretical references. Specifically, a mathematical task is proposed to explore and describe aspects of the mathematical thinking of the participating children when solving problems.

An open arithmetic problem was selected. This problem was implemented in a group of 23 students in pre-school (5-6 years) at a school in Cerdanyola del Vallès (Cataluña). The problem was solved by the children individually and in written form. In addition, to recognize the strategies and processes followed by the children in solving the problem, a semi-structured interview was conducted, and a video was recorded.

The problem is an adaptation of the one proposed by López (2015), where a family context is proposed to children, related to food. The following is the statement of the problem:

> You want to make a macedonia. You can buy bananas, pears, oranges and apples. In total, you buy 15 pieces of fruit. Explain how many pieces you buy for each fruit.

Different aspects were considered in the selection and adaptation of the problem, such as those pointed out by Baroody (1993) when characterizing the non-routine problems: the unknown is not obvious, the problem provides information on the total of fruits, but the unknown refers to the number of fruits of each type that could be used to make a macedonia with that total. It can be solved in different ways and different solutions can be obtained.

The problem was presented to the children orally and the following guidelines were given:

- The problem must be solved individually.
- Different representations can be used: drawings, numbers, letters or several of them at the same time.
- At least two types of fruit should be used to respond to the problem, not all four types are required.
- It must be considered that 15 pieces of fruits should be used

The children had blank sheets of paper to make their proposals. According to Edo and Marín (2017) at the moment of the proposal it is important to select the instructions appropriately so that the graphic representation the child makes shows what he/she thinks and how he/she thinks. In turn, it is desirable to create a climate of confidence and tranquility so that each child can reflect, choose, represent, and explain his or her reasoning. Blank paper marks will show languages and meanings, allowing the teacher to observe each student's learning and thus giving relevance to the student's marks (Carruthers and Worthington, 2006).

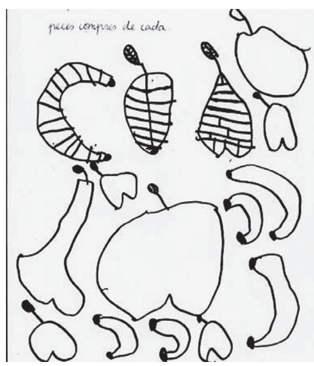
Regarding the semi-structured interview, a series of basic questions were set:

- Can you explain what you did?
- How many fruits of each type have you drawn? Why?
- Are you sure you have 15 fruits? How do you know?
- Have you drawn all kinds of fruit? Why?
- What have you done to know when you should stop drawing?
- Have you tried to use the same number for each type of fruit?
- What do the numbers you used indicate?
- How did you know how many more fruits you should draw while solving the problem?

The data from this research are the written protocols of each of the participants to the proposed task and the transcripts of the dialogs generated in the interviews. This information is initially organized into a data collection tool. As an example, Figure 2 shows an extract of that instrument. This includes the identifier assigned to the student (A1), the written answer and the initial part of the interview.

Figure 2

Extract from the data collection instrument

Student	Written Protocol	Transcription
A1		P: ¿Qué has realizado? Q: What have you done? A1: I've drawn them all Q: All? A1: Yes Q: Very good How many bananas have you made? A1: Four bananas Q: What else have you drawn? A1: This round thing are strawberries, there are two Q: Two strawberries? And this round, are these oranges? A1: Yes. They're oranges, huh! Q: How many have you done? A1: One, two, three, four, five, six, seven! Q: Teacher A: Student

Own elaboration.

Data analysis was carried out in two phases. In the first phase, the children's responses to the problem were individually analyzed, focusing this analysis on the *representations*. Following the proposal of Carruthers and Worthington (2005), the *representations* are classified into three categories: *iconic, written*, and *symbolic*. The two initial categories (dynamic and pictorial) proposed by these authors are not considered because of the age of the children participating in the study. Table 1 describes the indicators associated with each of these categories

Table 1

Indicators of the representation category

Categories	Indicators
Iconic	Uses a conceived picture of reality
Written	Uses letters or words to complete the answer
Symbolic	Includes numerals, dots, lines, circles, or signs

In the second phase to complement the study of children's productions, the *calculation methods* that followed in solving the problem were analyzed. For this analysis, continuing with the characterization taxonomy of children's mathematical graphs proposed by Carruthers and Worthington (2013), the categories of the dimension methods of written calculations were considered: *continuous counting, set separation, symbol scanning*, and *calculations with standard small number symbols*. The *large number calculation category* is not considered as it does not fit the problem conditions.

As mentioned Badillo *et al.* (2014), we also think that the solution and representation strategies raised by children are connected and, therefore, a global look will allow a richer analysis of the mathematical practices developed by children in solving the problem. An analysis instrument is constructed, combining the aspects analyzed in phases 1 and 2. Then in Figure 3 we illustrate the analysis of the answer provided by student ten (A10).

Figura 3

Analysis of A10

Student	Written Protocol	Transcript of the interview	Analysis
A10		Q: Tell me, what have you done? Have you drawn the fifteen pieces of fruits? A10: Mmm pears, I made a pear, with pears Q: You made many pears, and what else? Is this a banana? A1: A banana Q: I see that you have used num bers, was it to count what number each was until reaching fifteen? If you start with one, two, three, the eight… where is the fifteen? A1 Here Q: Have you gotten fifteen then? Did you stop when you reached fifteen? A1: Yes Q: Good job! Q: Teacher A: Student	*Representation* Student 10 performs a representation that can be classified as iconic and symbolic. On the one hand, with the representation the child shows the image he/she has of the fruits, and on the other he/she adds numbers to list each of the pieces. *Strategies* The student focuses on drawing to solve the problem. Represents the amounts that counts. The student draws all the fruits and focuses on reaching the final number (15) by counting each piece at a time, i.e., uses the ordinal by extension. It seems that the student recognizes the cardinal, and that uses the numeral with an order function; it is evident that the student is exploring with symbols.
Q: Teacher A: Student			

Own elaboration.

3 Results

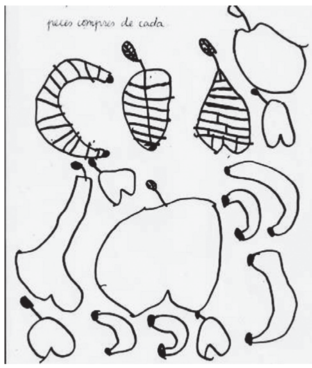
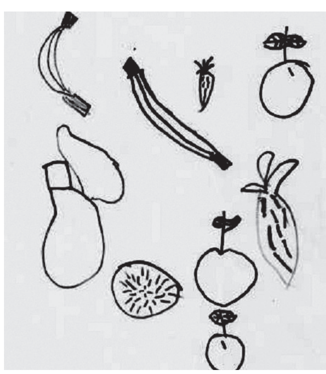
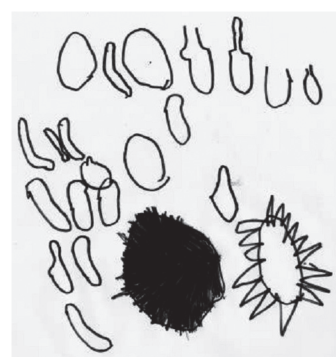
The results are organized into two parts: Characterization of the representations used by children and identification of the strategies developed in solving the problem.

3.1 Characterization of the representations used by children

As for the type of representations, 23 students who participate in the study use iconic representations. In using this type of representation, children rely on drawing to count and control the total amount defined in the proposed problem. At the same time, their drawings indicate the type of fruits chosen by children, the quantity they have considered for each type and in some cases the intentionality of grouping them (distributions). It should be mentioned that, although everyone proposes an iconic representation, not all adequately solve the problem, as in five of the students. Figure 4 shows different examples of responses, showing the above aspects.

Figure 4

Examples of different types of iconic representation responses

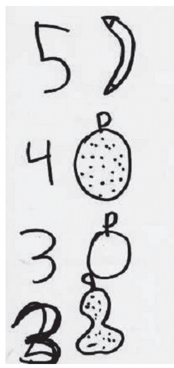
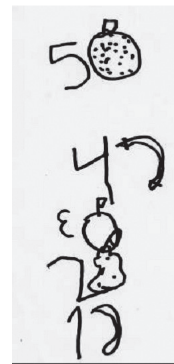
Correct problem answer-No grouping	Correct problem answer-With grouping
	
Representation of student 1 (A1)	Representation of student 2 (A2)
Inadequate answers to the problem	
	
Representation of student 14 (A14)	Representation of student 16 (A16)

Out of the total of children, three, in addition to using an iconic representation, also use a symbolic representation. In these representations a more complex reasoning is evident. Children who performed iconic-symbolic representations no longer focus only on counting to reach the total, but on the operational, proposing different subgroups to meet the condition of having 15 fruits in macedonia. These representations include, in addition to the drawing of the fruits of each child, the numerals that indicate the amount they have associated with each type (see figure 5).

Figure 5 shows an example of a response that combines representations. In the representations proposed by A4 and A12, it can be seen that children recognize that the whole (15 fruits) can be separated into discrete sets of various elements (e.g. bananas, oranges, apples and pears), which may (or may not) have different cardinal (e.g. 5, 4, 3, 3 or 5, 4, 3, 2, 1).

Figure 5

Examples of response of A4 and A12

Response with combination of iconic-symbolic representations	
	
Representation of student A4	Representation of student A12

3.2 Identification of calculation methods developed in problem solving

As for the methods used by children to solve the problem according to the dimensions proposed by Carruthers and Worthington (2006), 19 of the children's solution proposals were classified in the category of *continuous counting* and four in the category of *separation of sets*. In the first case, representations that express a quantity are classified. Children represent and account for things they choose (in this case fruits) but do not see physically (e.g. A1, see figure 4). Usually, this type of representation is drawn in a horizontal linear layout (e.g. A2, see figure 4), although others can be found. This is considered the first step in the calculation exploration. In our case, most of the children focused more on the goal of having 15 pieces of fruit than on the order in choosing which type of fruit to draw. If we look at the solution given by A10 (Figure 3) the final number of its count represents the total. These different mathematical practices related to continuous counting are key to recognizing the strategies children are developing (Carruthers and Worthington, 2006). Thus, some children, as the interviews showed, continually counted, starting with one of the fruits, what they considered "the first." Almost all children understand that it is necessary to count everything to get to a total, except those who failed to adequately solve the problem (e.g. A14, A16-Figure 4).

In the second case, the representations indicate separations in fruit subgroups (e.g. A2 - figure 4, A4 and A12 - figure 5). In our study, none of the children used marks (lines, words, circles, etc.) to differentiate the subgroups. But in the four cases in this category, the children represent separate sets (of fruits) that then add up to meet the condition of having 15 pieces in total in the macedonia. Student A2 (Figure 4) uses the space and distribution of the fruits to indicate the sub-groupings he has made of the fruits. The interview corroborates that he has made banana-apple-pear groups. Identifying the set and the elements of the set and repeating it continuously until reaching 15. In the case of A4 (Figure 5), it was possible to see in the interview that he performs the calculation as a narrative in words (Carruthers and Worthington, 2006) when describing what he has done: "I added five bananas and four oranges here, and then I added three apples and three pears to have a total of 15."

Finally, there are three solutions in the dimension: *symbol exploration*. According to Carruthers and Worthington (2006) children in this category organize their solutions and sometimes represent them leaving a space between the sets to imply that an (operant) is needed in that place, for example, as in the solution given by A4 (Figure 5). They usually use personal or invented symbols or approximations of standard symbols. In our study, the three children use standard numeral symbols to indicate the amount they associate with each type of fruit or to support continuous counting (A10-figure 3). In another case, one of the children (A12-figure 5) used the word "and" to indicate "+". The combination of drawings, words, numbers and/or personal symbols is also typical of this category, as is the case in the three cases mentioned above.

Children who perform representations combining the iconic and symbolic are those who also use symbol exploration strategies, demonstrating a more complex reasoning. Moving to other types of strategies, such as performing standard symbolic operations with small numbers, requires that children have developed them previously. However, as suggested by Vanegas and Giménez (2018), when children solve problems, the important thing is not to move from one strategy to another, but to use appropriate strategies that show an increasingly complete and adequate interpretation of the problem.

Discussion and conclusions

As suggested by Carruthers and Worthington (2005), we consider that children sometimes use a combination of representations, for example, iconic and symbolic, when they are in a period of transition. It seems that when they do it, they are moving from familiar representations to new ones, although they are not yet ready to dispense with non-essential elements. In our study, this occurs with three out of the 23 participants. This transition period is very important as children move toward more abstract forms of mathematics. However, it is also important to note that some children return to less developed graphic forms when they find that the mathematics presented is more challenging, because they are based on prior knowledge and ways they feel more confident.

When children move from recognizing numbers as symbols associated with different contexts in their lives to writing them for their own specific purposes, it evidences a significant change, because when they choose to write certain numbers (in our study, to indicate the total number of fruits of each type they would use to make macedonia) they have moved what they read into standard symbolic language and have chosen to use them in meaningful contexts (Worthington and van Oers, 2017). It is important to engage children in play and problem-solving environments that challenge them and allow them to experiment and choose their own methods.

Seeing the different representations children use when facing math tasks will allow teachers to better recognize their ways of thinking and the aspects they give relevance to when working with certain mathematical notions. In addition, the analysis of representations and associated strategies will allow the teacher to better evaluate the development of children's mathematical thinking. Consequently, new schoolwork can be designed to help children develop skills to explain, describe, relate and argue.

The richness of the problems and/or tasks proposed to children is a key element in enhancing the development of their mathematical thinking. Indeed, problems must be posed in a wide range of contexts that have real meaning for children, since it will help them have a personal sense of mathematics. As proposed by NAECY and NCTM (2013), we consider that problem solving, reasoning, communication, connections, and representation make it possible for children to acquire knowledge of the content. These processes develop over time if they are supported by well-designed learning opportunities. Children's development and use of these processes is one of the most enduring and important achievements of mathematics education. Their intuitive ideas become true mathematics when children reflect on them, represent them in different ways, and connect them to other ideas.

With this research we have been able to show how the analysis of children's representations and strategies in a problem-solving environment can provide important feedback on children's thinking; therefore, relevant elements for reflection on this subject in the initial training of early childhood education teachers. We hope to continue working in this topic, on the one hand, by exploring the representations that children construct when they engage in different mathematical tasks; on the other, by analyzing and using different references to characterize these representations and incorporating these findings into professional tasks in the initial training of teachers. We are interested that teachers understand that it is possible to develop quality mathematics from the early ages (Lee and Ginsburg, 2007) and to identify how research results such as the one described in this article can be useful in their professional context, supporting design, planning and evaluation of school proposals that promote the development of mathematical thinking in early childhood education.

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