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Didactic-Mathematical Knowledge mobilized by future mathematics teachers

Conocimiento didáctico-matemático movilizado por futuros profesores de matemáticas

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Abstract

From the perspective of mathematics education, several studies focus on the training of mathematics teachers. This article is a part of a master's research. It aims to analyze the didactic-mathematical knowledge mobilized by future mathematics teachers when solving tasks centered on themes related to financial education. It is qualitative research developed in the context of teacher education, consisting of a sample of undergraduate students in mathematics from two public higher education institutions in the North of Minas Gerais (Brazil). For data collection, a workshop was developed with five tasks, four of which were synchronous, carried out by Google Meet, and one asynchronous, posted on Google Classroom, which articulates mathematical content with themes related to Financial Education, namely: reflection on a class of math; reflecting on inflation and price indices; analyzing housing finance; analyzing interest and its applications in credit operations; elaborating activities related to interest applications. Data analysis, carried out from the mathematical and didactic dimensions of the Didactic-Mathematical Knowledge (DMK) model, revealed that future teachers satisfactorily mobilized the common knowledge of the mathematical content and, partially, the expanded knowledge. The mobilization of didactic knowledge was satisfactory only in the cognitive facet and partial in the other facets of the DMK.

Keywords: Mathematics teachers' formation, Didactic-Mathematical Knowledge, financial education.

Resumen

Desde la perspectiva de la Educación Matemática, varios estudios se centran en la formación de profesores de matemáticas. Este artículo forma parte de una investigación de maestría. Tiene como objetivo analizar el conocimiento didáctico-matemático movilizado por los futuros profesores de matemáticas al resolver tareas centradas en temas relacionados con la Educación Financiera. Se trata de una investigación cualitativa desarrollada en el contexto de la formación de profesores, constituida por una muestra de estudiantes de licenciatura en matemáticas de dos instituciones públicas de enseñanza superior del norte de Minas Gerais (Brasil). Para la recolección de datos, se desarrolló un taller con cinco tareas, cuatro de ellas sincrónicas, realizadas por Google Meet, y una asincrónica, publicada en Google Classroom, que articulan contenidos matemáticos con temas articulados con la Educación Financiera, a saber: reflexión sobre una clase de matemáticas; reflexión sobre la inflación y los índices de precios; análisis de la financiación de la vivienda; análisis del interés y sus aplicaciones en las operaciones de crédito; elaboración de actividades relacionadas con las aplicaciones del interés. El análisis de los datos, realizado a partir de las dimensiones matemática y didáctica del modelo de Conocimiento Didáctico-Matemático (CDM), reveló que los futuros profesores movilizaron satisfactoriamente el conocimiento común del contenido matemático., parcialmente, el conocimiento ampliado. La movilización del conocimiento didáctico fue satisfactoria solo en la faceta cognitiva y parcial en las demás facetas del CDM.

Descriptores: Formación de profesores de matemáticas, Conocimiento Didáctico-Matemático, Educación Financiera.

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1 Introduction

Teachers play a vital role in the development of the teaching and learning processes of mathematics at different educational levels, which requires training that allows them to relate the knowledge of the content to be taught with didactic and methodological strategies that would overcome possible obstacles derived from educational processes.

In the teaching of Mathematics and Natural Sciences, teacher training is a relevant topic studied in different research (Gellert *et al.*, 2012; Ponte, 2014; Stahnke *et al.*, 2016; Potari and Ponte, 2017, Barros *et al.*, 2014). More specifically, one of the research trends is focused on specialized mathematical content knowledge for teaching. Godino *et al.* (2017), state that such knowledge has been discussed by several authors through different theoretical perspectives and that in the specific literature related to mathematics teacher education is:

> It is recognized that the didactic training of teachers is a field of scientific and technological research that requires attention from the Didactics of Mathematics, since the development of students' thinking, and basic mathematical skills depends on such training. (Godino *et al.*, 2017, p. 91)

Considering the relevance of the discussions around the necessary knowledge for the teacher (Breda *et al.*, 2018; Giacomone, 2018; Morales-Maure, 2019), this article aims to analyze the didactic-mathematical knowledge mobilized by future mathematics teachers when solving tasks focused on topics related to Financial Education from the perspective of Mathematics Education, whose relation will be described below.

The teaching practice that seeks the approximation between mathematical contents and reality through a critical and reflective perspective highlights the need to relate mathematics with contextualized topics, which can be enhanced through an approach to specific topics of Financial Education, especially related to the current economic scenario. Reinforcing this idea, the Survey of Consumer Indebtedness and Delinquency (PEIC), conducted by the National Confederation of Commerce of Goods, Services and Tourism-CNC (2021, p. 1, the translation is ours) in Brazil, shows that "the percentage of families with debts in the country [Brazil] ended in 2020 at a high level, after three consecutive months of reduction". Similarly, research proposes deepening discussions on Financial Education in all social sectors, especially in schools, given that money management begins in childhood (Teixeira, 2015; Souza, 2018; Assis, 2019; Martins, 2019; Melo, 2019; Ferreira, 2020), which can be achieved through School Financial Education (Silva and Powell, 2013).

We corroborate the idea that highlights the potential of approaching Financial Mathematics contents, contextualized and coherent with the reality of students, both to work School Financial Education from the beginning of Elementary school (Teixeira, 2015) and to instrumentalize financial literacy, which is directly associated with:

> The ability to read, analyze and interpret financial situations; knowledge of the essential and necessary elements of financial mathematics relevant to the context of the subjects; ability to take a reasoned critical stance; ability to consider the variables and implications of their actions; make conscious decisions oriented towards individual and social financial well-being. (Sena, 2017, p. 58, the translation is ours)

The research, approved by the Research Ethics Committee of the State University of Montes Claros (Unimontes) through the Certificate of Ethical Appreciation (CAAE) No. 30562920.6.0000.5146 and Opinion No. 4.031.671, consists of qualitative research which, according to Godoy (1995, p. 21, the translation is ours), seeks "to capture the phenomenon studied from the perspective of the people involved,

considering all relevant points of view". The sample consisted of twelve undergraduate mathematics students from two public institutions in northern Minas Gerais. Data collection was performed by recording meetings in *Google Meet*, joint solutions and discussions of the proposed tasks, and the participants' productions in *Google Classroom*. The data analysis focused on the categories included in the Didactic-Mathematical Knowledge model, developed by Godino (2009) and Pino-Fan and Godino (2015).

The following sections deal with the synthesis of the theoretical framework, the methodology, the analysis and discussion of the data and the final considerations.

1.1 Theoretical framework

This research focuses on the theoretical tools developed in the context of the onto-semiotic approach to mathematical knowledge and instruction-EOS. (Godino *et al*., 2007).

According to the EOS, the mathematics teacher must have a mastery of mathematical content far beyond the knowledge that must be mobilized by the students of the educational level in which he/she is going to work. However, knowledge mobilization of teachers does not guarantee a convincing performance in the teaching and learning processes, since these processes are complex, so it is essential to have a deeper knowledge of both mathematics and its didactics, beyond the knowledge manifested/mobilized by students.

To investigate this knowledge in the context of mathematics teacher education, the Didactic-Mathematical Knowledge (DMK) model (Godino, 2009; Pino-Fan and Godino, 2015) of the EOS was developed, which is articulated with other models related to teacher knowledge, such as Pedagogical Content Knowledge (Shulman, 1986, 1987), Mathematical Knowledge for Teaching (Hill *et al*., 2008) and the notion of Competence (Shoenfeld and Kilpatrick, 2008).

This model interprets and characterizes the teacher's knowledge, which is approached in this research from two dimensions: mathematical and didactic.

The mathematical dimension refers to the specific knowledge of mathematics, including the subcategories *common knowledge and extended content knowledge*, as pointed out by Carvalho (2017). The first refers to what the teacher needs to know about a specific mathematical object to solve problems and tasks proposed in textbooks and other material and technological resources at a given educational level (shared knowledge between teacher and student). On the other hand, the second allows the teacher to establish a relationship between the mathematical object of study and other mathematical notions at the same educational level and higher levels.

The didactic dimension includes the six categories of analysis described below.

Epistemic: related to the didactic-mathematical knowledge of the teacher; it is the understanding of mathematics for its teaching, it allows to perform different representations of a mathematical object, to solve a task by different methods, to establish relationships between the mathematical object of study and other mathematical objects of previous and subsequent educational levels. This integrates both the notions of the proficiency model (Schoenfeld and Kilpatrick, 2008) about knowing mathematics in depth and the ideas of specialized content of the content (Hill *et al*., 2008).

Cognitive: referred to the knowledge of how students learn mathematics, which allows establishing a relationship between personal meanings (student's knowledge), institutional meanings (knowledge from the point of view of the educational center) and class planning, taking into account the foresight of possible errors and difficulties.

Affective: inherent to the affective and emotional factors of students with respect to mathematical objects. It has to do with the knowledge necessary to assimilate the students'

moods and the factors that motivate or not the resolution of a mathematical problem. Both the cognitive and affective factors incorporate and expand the notions of knowledge about the student and his or her characteristics (Shulman, 1987), about the knowledge of students as people who think and learn (Schoenfeld and Kilpatrick, 2008), and about the knowledge of the content and of the students (Hill *et al.*, 2008).

Interactional: it is related to the knowledge that allows predicting, implementing and evaluating the moments of interaction between teachers and students, among students, among others that occur in the teaching and learning processes. This aspect considers the ideas of building relationships that support learning, proposed by the notion of competence for teaching (Schoenfeld and Kilpatrick, 2008), which involves knowledge to predict, implement and evaluate sequences of interactions carried out by teachers and students involved in the teaching and learning process.

Mediational: refers to the teacher's knowledge regarding the use of material and technological resources, as well as the allocation of time to improve the learning of mathematical content.

Ecological: refers to the teacher's knowledge of interdisciplinarity, the mathematics program and the social, political and economic contexts.

The aspects that make up the didactic dimension make it possible to analyze, describe and develop the knowledge of the teacher or future teacher involved in the different phases of the teaching and learning process. Faced with a mathematical task, the teacher must be able to mobilize the diversity of meanings that arise from it, presenting different solutions from various representations, explanations, and justifications, in addition to promoting adaptations to the students' level of knowledge (Godino *et al.*, 2017).

2. Methodology

This research is qualitative, following the proposal of data collection and analysis. The data were analyzed through content analysis (Bardin, 2016) based on the categories proposed by DMK.

According to Bardin (2016), content analysis is organized in three stages, pre-analysis, exploration of the material, processing of the results, inference and interpretation.

Pre-analysis aims at choosing and elaborating indicators. In turn, the exploration of the material consists of coding, decomposition or enumeration procedures, based on pre-established rules. Data processing seeks a condensation of the important information from which inferences will be made.

The data were collected from the development of a workshop on the digital platform *Google Meet,* through five remote meetings, lasting four hours each, for a sample of twelve undergraduate students in mathematics, during January 2021.

For the conduction of the workshop, four synchronous tasks and one asynchronous task were proposed, which relate mathematical contents with the following topics related to Financial Education:

- Task 1: Reflection on a mathematics lesson.
- Task 2: Reflection on inflation and the HICP and CPI price indexes.
- Task 3: Analysis of housing finance in the national context.
- Task 4: Analysis of interest and its applications in credit operations.
- Task 5: Development of activities related to the applications of interest rates.

The synchronous tasks (1 to 4) were developed in groups, with subsequent socialization and discussion of answers, while task 5, asynchronous, was carried out individually, with subsequent socialization. Transcriptions of the recordings were prepared and used in the analysis and discussion of results, together with the other activities developed by the participants and published in *Google Classroom.* The data analysis was carried out based on the dimensions and aspects in the DMK and/or adapted from Godino (2009).

3 Results and discussion

The analysis of the data collected in the workshop was carried out based on the mathematical and didactic dimensions of the DMK model. In this sense, we present the analysis of the data through the aspects/indicators presented in the dimensions.

3.1 Mathematical dimension

The mathematical dimension of the DMK will be analyzed through *common knowledge*, referring to the mathematical knowledge mobilized by the future mathematics teachers and related to the mathematical contents presented in the proposed tasks; and *extended knowledge,* referring to the mathematical knowledge mobilized by the future teachers and related to more advanced levels of study, intramathematical relation of contents and generalizations.

To analyze common and extended knowledge, we used as a reference Task 4, which consisted of 21 items, including three problem-situations that enables to investigate common and extended knowledge. We used the following indicators:

Common knowledge: Mobilizes mathematical knowledge correctly, partially, or inadequately to solve tasks.

Extended knowledge: Generalizes when solving tasks and connects them to other more advanced content.

The common knowledge of the content consists of the mathematical knowledge of the future teacher related to the same level at which he/she would teach (Elementary and High School). It will be analyzed through the situations-problem 1, 2 and 3 of Task 4, the first two being adapted from the National High School Exam (*ENEM: Exame Nacional do Ensino Médio*), which require the mobilization of mathematical knowledge related to Financial Mathematics, whose topics are most frequently presented in activities related to Financial Education (Gaban, 2016; Martins, 2019). The first problem-situation addressed real estate financing, as explained in Figure 1.

Figure 1

Situation-problem 1

(ADAPTED FROM ENEM - 2015) A couple makes a real estate financing of R$ 180 000.00 to be paid in 360 monthly installments, with an effective interest rate of 1% per month. The first installment is to be paid a month after the disbursement, and the amount of the monthly installment is R$500.00 plus 1% interest on the remaining balance (amount due before payment). Note that with each payment the remaining balance is reduced by R$500.00 and consider that there is no installment due. Making the installment payments regularly, we ask for:

a) The detailed calculation of the amount of the installment paid in the 10th month, using three different strategies.

b) Name the mathematical concepts used in solving this situation-problem.

c) Identify the properties and/or theorems used in the different solutions of the situation-problem 1

d) Justify your answer with a logical argument

Note. Research data.

All groups correctly mobilized the knowledge necessary to find the solution to problem-situation 1. The other items will be discussed later in the epistemic aspect of the didactic dimension.

The first part of problem-situation 2 of Task 4, described in Figure 2, deals with credit operations, and will be discussed below.

Figura 2

Situation-problem 2 of task 4

(ADAPTED FROM ENEM - 2000) Juan wants to buy a car whose cash price is R$21,000.00 and this value will not be readjusted in the following months.
He has R$20,000.00 that can be used at a compound interest rate of 2% per month, and he chooses to leave all his money applied until the amount reaches the value of the car.
a) ( ) two months, and he will have the exact amount
b) ( ) three months, and he will have the exact amount
c) ( ) three months, and he will have approximately R$225.00
d) ( ) four months, and he will have the exact amount
e) ( ) four months, and he will have approximately R$430.00

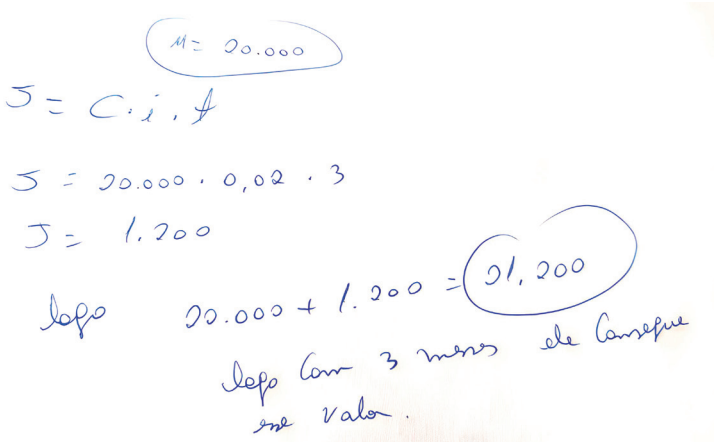
2.1. Solve the situation-problem 2, presenting all calculations and procedures used.
[Recall the formula $Mn=C\cdot(1+i)^n$, where Mn: amount in period n; C: capital; i: rate; n: period].
.

Note. Research data.

We note that all groups marked the correct alternative for solving problem 2. However, when analyzing the different solutions presented for item (2.1), we identified that Group 4 made a mistake by presenting a solution using simple interest, which is not coherent with the solution of the proposed problem-situation related to the compound capitalization system, which can be seen in Figure 3.

Figure 3

Solution of the situation-problem 2 by group 4



Note. Research data.

As this is an objective topic, the resolution proposed by Group 4, although it used an erroneous procedure, allowed them to find a value close to the correct alternative. We expected the research participants to arrive at the correct result by calculating the amount month by month or by applying the compound interest formula considering time as a variable.

The extended knowledge of the content will be analyzed from items 2.3 of situation-problem 2 and in situation-problem 3 of task 4, which include the deduction of a formula to calculate the amount and the resolution of a problem related to the capital accumulation factor, with the subsequent deduction of the formula, as shown in Figures 4 and 5.

Figure 4

Situation-problem 2 of task 4

Situation-Problem 2 (2nd part)
(ADAPTED FROM ENEM - 2000) Juan wants to buy a car whose cash price is R$21,000.00 and this value will not be readjusted in the following months.
He has R$20,000.00 that can be used at a compound interest rate of 2% per month, and he chooses to leave all his money applied until the amount reaches the value of the car. [...]

Note. Research data.

Figure 5

Situation-problem 3 of task 4

(ADAPTED-MATEMÁTICA FINANCEIRA) Determine the value of the amount, at the end of the fifth month, of a series of 5 monthly installments, equal and consecutive, in the amount of R$ 100.00 each, at a rate of 4% per month, knowing that the first installment is applied at the end of the first month, i.e., 30 days from the date taken as base (zero amount), and that the last one, at the end of the fifth month, agrees with the time when the amount is requested. [...]
3.2 Derive the expression (formula) to calculate the monthly installments.

Note. Research data.

Problem-situation 3 was correctly solved by groups 3 and 4. However, only the students of Group 4 were able to deduce the formula proposed in item 3.2, which shows that future mathematics teachers have difficulties in the deduction of formulas and mathematical generalizations. However, although the prospective teachers mobilized common content knowledge related to simple and compound interest, and the sum of terms of geometric progression, the results of this research show a low mobilization of extended knowledge.

On the other hand, we corroborate the finding of Coutinho and Teixeira (2015) that teachers do not have adequate training in Financial Mathematics, making difficult the insertion of Financial Education in Elementary school.

3.2 Didactic dimension

The didactic dimension, which refers to knowledge for teaching mathematics, will be analyzed through the six aspects of the Didactic-Mathematical Knowledge model: epistemic, cognitive, affective, interactional, mediational, and ecological. Each aspect will be synthesized using the following indicators:

Epistemic: Solves tasks using different representations, uses concepts and properties, and justifies task solutions.

Cognitive: Describes possible learning conflicts when students solve tasks.

Affective: Describes strategies that motivate students to perform the task.

Interactional: Presents methods to implement sequences of interactions among the agents involved in the teaching and learning process.

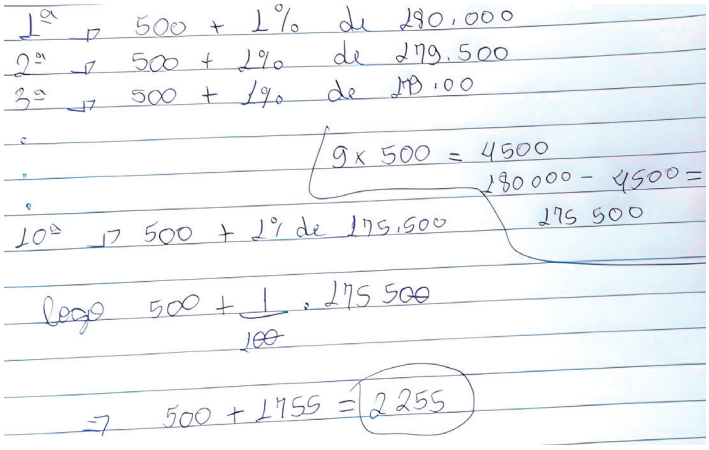
Mediational: Values the adequacy of time and the contribution of the use of materials and technological resources.

Ecological: Identifies the elements of the program that are addressed in the tasks and the social, material, and other factors that condition their performance.

The epistemic aspect covers the knowledge of the mathematical content required for teaching at the specific school level. This research includes tasks related to the specific objects of study of Financial Mathematics. The analysis of this aspect revealed that all groups were able to find the correct solution for the proposed problem-situation. However, they presented difficulties related to the elaboration of the solutions through different representations, predominating in all groups the solution of Group 2 shown in Figure 6.

Figure 6

Common procedure of all groups



Note. Research data.

Items b, c and d are related to the identification of mathematical concepts, properties and theorems, and the logical argumentation that justifies the answer. Although all groups found the correct solution to problem-situation 1, there was a generalized difficulty in these items.

According to Pino-Fan and Godino (2015), the epistemic aspect refers to the teacher's ability to solve a mathematical task through different procedures, identify the mathematical concepts that arise during the solution and relate the mathematical object addressed with others of the same educational level and more advanced. Corroborating these ideas, Carpes and Bisognin (2020) consider the teacher's ability as one of the essential tools for the development of student thinking and learning, since it allows identifying the emergent knowledge or prerequisites necessary to solve a task.

The cognitive aspect allows analyzing the knowledge of the future teacher related to the learning and mathematical understanding of secondary and high school students, being analyzed through tasks 2, 3 and 4. From the answers submitted by the groups, it was possible to identify the predominance of three types of errors or difficulties: (1) lack of understanding of the main concept addressed by the task and its application; (2) poor prior knowledge of the mathematical concepts necessary to perform the task; and, (3) inadequate use of digital technologies.

We can infer that the possible errors and difficulties expressed by the research participants may be because these tasks were contextualized with everyday issues and required both Financial Education and Financial Mathematics. Task 2 addresses problem-situations related to inflation and indexers; task 3 focuses on simulations through the *Caixa Econômica Federal* website and discussions related to decision making on real estate financing; and task 4 deals with interest and credit operations. We understand that one of the possible reasons why future teachers considered the lack of understanding of the concepts by elementary and high school students as an obstacle may be associated with their low affinity with the topics addressed in the proposed activities. As for the second error/difficulty indicated, all the tasks focus on content related to Financial Mathematics, especially percentages and simple and compound interest, which would require previous knowledge, from elementary and high school students, necessary for the development of the tasks, which may

not be well consolidated. Regarding the third error/difficulty mentioned, all the workshop tasks were developed through distance learning and, in addition, the completion of tasks 2 and 3 required access to the Internet and the manipulation of some technological resources, such as the Broad Consumer Price Index (IPCA) calculator and the *Caixa Econômica Federal* housing simulator, which leads us to interpret that these possible difficulties could be related to the lack of skills of the teachers to access and manipulate specific technological applications and resources.

The affective aspect deals with the teaching knowledge related to the emotional, affective, and attitudinal aspects of students in relation to mathematical objects. Some studies (Machado *et al*., 2010; Zan *et al*., 2006) mention the existence of a significant correlation between affectivity and cognition, in the sense that affectivity emanates from cognition, and cognition is part of affectivity. The participants of the research highlighted Financial Education as a "subject" that favors learning, because it allows the student to approach the subject of study with situations of their everyday life, giving mathematical contents a less abstract perspective, highlighting the relevant nature of mathematical knowledge by enabling the applicability of mathematics in situations that allow the individual to understand and solve everyday situations (Seah and Bishop, 2000; Campos *et al.*, 2015, Ribeiro *et al.,* 2018).

The interactional aspect considers the necessary professional knowledge about the interactions between the agents involved in the teaching process. Analyzing this aspect, we found that participants pointed out the relevance of developing activities in groups or in pairs, arguing that discussions among students would enhance learning. In this sense, Brandão and Neres (2018) point out a fundamental role of the teacher in the development of activities that promote cooperation and collaboration of all individuals involved in the educational process to provide a more attractive environment and enhance the cognitive development of the student.

Regarding the use of material and technological resources, as well as the adequacy of time for the development of tasks, aspects included in the mediational aspect, future teachers emphasized the need to adjust the workload in a way that allows to review mathematical contents related to previous knowledge, and the teacher preparation for a satisfactory use of technological resources that should be available to meet school demands.

Below are some excerpts taken from the collective discussion that took place during the second meeting, mentioning some concepts related to the preparation for the use of technological resources:

> A10: I do not feel prepared to teach mathematics with technologies; I consider that what I was taught during my training is not enough. Even if the school where I will work provides me the resource, I would not feel prepared.

> A1: I agree with A10, I feel prepared only to teach a quality class using GeoGebra or Excel because they were the only *computer programs* we used during my undergraduate studies. I believe that the approach of activities related to technological resources in higher education institutions is insufficient to prepare undergraduate mathematics students to use them adequately in their classes.

Cardoso and Figueira Sampaio (2019) state that, even though technology is part of the daily life of a considerable part of the population, there are still obstacles to its use in the classroom. One of the reasons is the deficiency in the use of technological resources in the initial training of teachers.

The *ecological aspect* considers the knowledge of the program that includes the object of study of mathematics at the corresponding educational level, its relationship with other programs and the social, political, and economic factors (Pino-Fan and Godino, 2015).

To reflect on the knowledge related to this aspect, the participants of this research were asked about the organization of topics related to Financial Mathematics throughout the levels of schooling and the approach to Financial Education in the Mathematics program of elementary and high school. Regarding the knowledge of the mathematical contents included in the mathematics program related to Financial Mathematics, all groups expressed a satisfactory knowledge. However, with respect to the insertion of Financial Education in elementary and high school Mathematics classes, Groups 1 and 4 did not express any opinion, Group 3 was unable to give an opinion and only Group 2 expressed a relatively consistent position.

The analysis of the ecological aspect of the didactic dimension of the DMK reveals that the future mathematics teachers satisfactorily mobilized knowledge about Financial Mathematics. However, they were not able to adequately mobilize knowledge about the insertion of Financial Education in the Mathematics academic program or establish its relationship with other mathematical or interdisciplinary contents. This enables us to identify the need to develop training processes focused on Financial Education in the context of the training of teachers who teach mathematics.

4. Conclusions

This article is part of a research conducted in the context of a master's degree, with the objective of analyzing the Didactic-Mathematical Knowledge (DMK) mobilized by future mathematics teachers when solving tasks focused on topics related to School Financial Education.

In line with the objective, this research allowed us to inquire into a formative process focused on the contents of Financial Mathematics through the contextualized approach of contents related to real situations (Teixeira, 2015) for future mathematics teachers.

Regarding Critical Mathematics Education, both teachers and future teachers assumed a fundamental role in the teaching and learning processes of the concepts of Financial Mathematics included in the different tasks developed from the perspective of School Financial Education, which contributed to the training of future mathematics teachers for the development of Financial Education at school (Campos *et al.*, 2015).

The analysis of the mathematical dimension of DMK revealed that the future mathematics teachers correctly mobilized common content knowledge by presenting correct solutions to all the selected questions, except for Group 4, which partially mobilized mathematical knowledge and made a mistake in the solution of situation-problem 2, and Groups 1 and 2, which presented incorrect solutions to situation-problem 3 of Task 4. As for the extended dimension of mathematical knowledge, its analysis was performed based on the tasks mainly related to formal deductions and generalizations, pointing to low mobilization. The results showed the difficulties of prospective teachers in moving from elementary to advanced mathematical thinking (Tall, 2002).

Regarding the didactic dimension of DMK, the research sought to analyze the didactic knowledge mobilized by future mathematics teachers based on indicators related to the epistemic, cognitive, affective, interactional, mediational, and ecological aspects. Based on the indicators used, we consider that didactic knowledge was mobilized satisfactorily in the cognitive aspect and partially in the other aspects.

The results of this research point out the potential of the DMK to carry out research focused on the analysis, categorization, production and/or mobilization of Didactic-Mathematical Knowledge in the context of the training of future mathematics teachers. Furthermore, they allow considering as perspectives for future research the development of studies related to the phases of the didactic design oriented to the mobilization of DMK of future

mathematics teachers on Financial Education, as well as the study and adaptation of suitability criteria to the complexity of Financial Mathematics objects included in training processes and/or activities focused on Financial Education.

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