ALTERIDAD
REVISTA DE EDUCACIÓN
p-ISSN:1390-325X / e-ISSN:1390-8642
http://alteridad.ups.edu.ec

july-december 2022
Vol. 17, No. 2, 166-179
https://doi.org/10.17163/alt.v17n2.2022.01

Influence of the teaching context on pattern representation in early childhood education

Influencia del contexto de enseñanza en la representación de patrones en educación infantil

Yeni Acosta is a professor at Universidad de Girona, Spain (yeni.acosta@udg.edu) (https://orcid.org/0000-0001-9873-2127)
Dr. Ángel Alsina is a professor at Universidad de Girona, Spain (angel.alsina@udg.edu) (https://orcid.org/0000-0001-8506-1838)

Received on: 2022-02-14 / **Revised on:** 2022-05-27 / **Accepted on:** 2022-06-07 / **Published on:** 2022-07-01

Abstract

According to the Mathematics Teaching Itineraries Approach (EIEM), which proposes intentional teaching sequences from the concrete to the symbolic, we analyse how the teaching context influences the repetition pattern tasks in a group of 24 children during two consecutive school years (4-6 years old). For this purpose, repetition pattern tasks have been implemented in the two extreme contexts of a previously designed and validated itinerary: real situations and graphic contexts, respectively. The data have been analysed from ethnographic methodological schemes of participant observation (field diary); pedagogical documentation (audiovisual record); and written productions of the patterns (representations). The main results obtained show that: a) in pupils aged 4-5 years, a positive difference of 32.9% of real situations versus graphic resources has been identified; b) in pupils aged 5-6 years, although the difference between the two contexts decreases slightly, it continues to be above 30%. It is concluded that the teaching context influences the understanding of repetition patterns, so that it is necessary to teach patterns from the situational to the formal level.

Keywords: Repetition patterns, representation in mathematics, real situations, graphical resources, early childhood education.

Resumen

Con base en el Enfoque de los Itinerarios de Enseñanza de las Matemáticas (EIEM), que propone secuencias de enseñanza intencionadas desde lo concreto hasta lo simbólico, se analiza cómo influye el contexto de enseñanza en las tareas con patrones de repetición en un grupo de 24 escolares españoles durante dos cursos académicos consecutivos (4-6 años). Para ello, se han implementado tareas de patrones de repetición de los dos contextos extremos de un itinerario previamente diseñado y validado: situaciones reales y contextos gráficos, respectivamente. Los datos se han analizado a partir de esquemas metodológicos etnográficos de observación participante (diario de campo); la documentación pedagógica (registro audiovisual); y las producciones escritas de los patrones (representaciones). Los principales resultados obtenidos muestran que: a) en el alumnado de 4-5 años se ha identificado una diferencia positiva del 32,9 % de las situaciones reales frente a los recursos gráficos; b) en el alumnado de 5-6 años, si bien desciende ligeramente dicha diferencia entre ambos contextos, continúa estando por encima del 30 %. Se concluye que el contexto de enseñanza influye en la comprensión de los patrones de repetición, por lo que es necesaria una enseñanza de los patrones desde el nivel situacional hasta el formal.

Descriptores: Patrones de repetición, representación en matemáticas, situaciones reales, recursos gráficos, educación infantil.

Suggested citation: Acosta, Y. and Alsina, Á. (2022). Influence of the teaching context on pattern representation in early childhood education. *Alteridad, (17)*2, 166-179. https://doi.org/10.17163/alt.v17n2.2022.01

1 Introduction and state-of-the-art

Attention has been addressed to children in recent years for the integral development of the person. Bowman *et al*. (2001, p. 23), state that "[…] young children are capable students and that the educational experience during the preschool years can have a positive impact on school learning". In this sense, research suggests that early mathematical competencies (basically 4 to 6 years old) can be an indicator of academic success in later stages (Nguyen *et al.,* 2016; Rittle-Johnson *et al.,* 2017; Wijns *et al*., 2021, among others). Therefore, effective planning and implementation is necessary to promote the development of mathematical competence from the earliest ages, diversifying the use of educational scenarios and leading to good practices in the mathematics classes (Alsina, 2019, *National Council of Teachers of Mathematics* [NCTM], 2000, 2014).

From this perspective, this article assumes the focus of the Itineraries of Teaching Mathematics in Alsina (2019, 2020), henceforth EIEM, that states that the teaching of mathematics in the early ages should be proposed as a journey from the concrete to the abstract, through teaching sequences that contemplate different teaching contexts to promote a consolidated acquisition of mathematical competencies. This approach establishes an intentional sequence that includes three teaching levels that go from the particular to the general, offering a hierarchical orientation of the contexts that make it up. At the first levels are the informal contexts that should be "consumed" on a daily basis: real situations, manipulative resources, and the use of recreational proposals; in the next level, reference is made to intermediate contexts that should sometimes be used, such as literary and technological resources; and finally, formal contexts that should occasionally be implemented, such as tabs and textbooks.

This article applies this approach to the teaching of repetition patterns, as it has been shown that knowledge of patterns and their structure positively influences the early development of mathematical thought (Clements and Sarama, 2015; Lüken and Kampmann, 2018; Mulligan *et al.,* 2020; Papic *et al*., 2011; Rittle-Johnson *et al*., 2018; Tirosh *et al.,* 2018; Wijns *et al*., 2021). Therefore, pattern exploration can be seen as a gateway to promoting generalization (Vanluydt *et al.,* 2021), anticipation, justification, and the representation and precise use of mathematical language (Acosta and Alsina, 2020). When speaking of mathematical patterns, it is necessary to distinguish between pattern as an ordered sequence and between pattern structure, i.e., organization or rule underlying the pattern (Mulligan and Mitchelmore, 2009). These Australian authors point out that patterns comprise two components: one cognitive, related to knowledge of structure; and another meta-cognitive, associated with the ability to search and analyze patterns. Mulligan *et al*. (2020) say that the lack of consciousness of the pattern and its structure may become a predictor of future mathematical difficulties. However, little is known about how the teaching context influences the understanding and representation of patterns. In fact, authors such as Wijns *et al.* (2019) mention the need to study whether tasks with patterns that are implemented optimally promote their full potential, thus fostering the development of algebraic perceptions of schoolchildren. As Alsina (2020) states, the textbook is sometimes seen as a preponderant resource that leaves no room for addressing mathematical concepts and procedures from other scenarios that are more realistic, concrete, and meaningful to schoolchildren.

Hence, the aim is to provide data that allow an approximation to the way in which children from 4 to 6 years old perform repetition patterns and exteriorize their representation in different teaching contexts, in order to address the development of the understanding of patterns in a contextualized and longitudinal way, considering the approaches of the EIEM.

From this perspective, we raised the following research questions:

- How does the teaching context influence tasks with repetition patterns?
- What is the relationship between concrete teaching contexts (real situations) and abstract teaching contexts (graphic resources) during the understanding and representation of repetition patterns?

The objectives are:

- Analyze the relationship between understanding and representing patterns of repetition.
- Demonstrate the influence of the learning context on the success of representing repetition patterns

1.1 Teaching and representation of repetition patterns from the approach of didactic itineraries

The baseline of EIEM as a theoretical framework of our study are discussed; it is defined what is understood by pattern, the importance of its teaching and the representation as a mathematical process that promotes understanding.

Acosta and Alsina (2021) point out that the learning of patterns begins in concrete situations until it is consolidated into abstract experiences. Therefore, the EIEM (Alsina, 2019, 2020) is taken as a reference, based on three interrelated bases: a) the sociocultural perspective of human learning (Vygotsky, 1978), understanding education as a social and cultural phenomenon that sees language and interaction as essential tools to promote learning; b) The realistic model of teacher training (Korthagen, 2001), which considers that teachers should be familiar with various ways of intervening and exercising them in practice, i.e., they should have criteria for knowing when, what and why a situation is likely to reflect systematically; and c) Realistic Mathematical Education (Freudenthal, 1991), which promotes the use of contextualized problems in real situations as the beginning of the teaching-learning process of mathematics.

Based on these bases, the EIEM (Alsina, 2019, 2020) considers the teaching of mathematics through didactic sequences that include the following three levels:

- Informal level: The teaching of mathematical contents is prioritized from contexts of real situations and close to the students, using manipulative and recreational materials, relying, in turn, on informal knowledge, common sense and experience. In these contexts, the cognitive demands that are used are: exploration, manipulation, or experimentation, conforming as requirements to visualize and understand mathematical ideas in a concrete way.
- Intermediate level: The teaching of content continues into contexts that are formed as a bridge between the contexts of the previous phase and the formal contexts of the later phase. In this level are literary (stories and songs) and technological resources (Applets, programmable educational robots, etc.). In these contexts, cognitive demands are focused on exploration and reflection, which progressively facilitate the schematization and generalization of mathematical knowledge.
- Formal level: The teaching of content ends in graphic and symbolic contexts, where the representation and formalization of mathematical knowledge is encouraged, using conventional procedures and notations to promote the learning of the concrete to the symbolic. Therefore, cognitive demands in these contexts are mainly focus on abstraction and generalization.

From this approach, the target is more in heuristic activities rather than pure exercise, and critical mathematical thought more than repetition (Alsina, 2019).

As indicated in the introduction, this study focuses on the teaching of patterns. When speaking of pattern, we refer to a sequence of elements ordered according to a given norm, rule, nucleus, or periodic unit. Clements and Sarama (2015) explain that the teaching of patterns pursues the search for regularities and mathematical structures. Recognizing patterns is set as a fundamental capacity for many domains of knowledge such as reading, mathematics, or arts, since patterns provide meaning and cohesion (Björklund and Pramling, 2014). For this reason, Papic (2015) suggests the need to promote awareness among boys and girls about patterns to stimulate structural development, relational understanding, and generalization from an early age, and lay the baselines of mathematical thinking in general and algebraic. A consolidated algebraic thought requires the capacity to symbolize and generalize (Sibgatullin *et al.,* 2022).

Our study assumes that patterns can vary according to their regularity and content; based on this statement, patterns can present units that are repeated, that are arranged in a structural or symmetrical way or that grow (Bock *et al*., 2018). The typology of patterns addressed in our teaching itinerary is repetition, i.e., patterns that through iterative sequences show regularities or repetitions of specific qualitative and/or quantitative characteristics (color, shapes, size, sounds, or numbers, e.g., "green, green, yellow, green, green, yellow" or "■○■○").

Prestigious authors and institutions point out that the teaching of repetition patterns and the understanding of their structure positively influences early mathematical development, as it promotes a truthful baseline for algebraic thinking (Mulligan *et al*., 2020; Rittle-Johnson *et al*., 2018; Wijns *et al*., 2019). Developing the concept of pattern involves perceiving the underlying rule and consciously and functionally identifying the unit of repetition. According to Wijns *et al*. (2019), it is necessary to implement tasks with patterns that give children the opportunity to transition from recursive to functional thinking, i.e., to observe the relationship of elements that lies in a series to abstract and represent the internal structure in a guided way.

What does representation in mathematics imply? For Freudenthal (1991) the progressive development of the representation of mathematical ideas and procedures goes from the concrete to the abstract, so that it can take diverse forms through physical objects, natural language, drawings, and conventional symbols. Reed says (2001, p. 215), "drawing can be a window into a child's mind". Therefore, it is necessary to respect and encourage the process of representation to learn (and to understand) the symbol that represents an object, a situation, or a mathematical idea. For this reason, Duval (1995, p. 15) considers that "there is no knowledge that can be transferred by an individual without a meaningful activity". Also, in relation to representation as a mathematical process that externalizes student understanding, Pino-Fan *et al*. (2017) says that such a process plays an essential role in acquiring and treating an individual's knowledge. From this perspective, the NCTM (2014) is committed to teaching mathematics in a way that enables connections between representations to be established to effectively link conceptual and procedural understanding.

Considering the latter, we conceptualize the representation in mathematics as an interconnected process that allows to express in a concrete way the knowledge and mathematical procedures that students possess using different signs, graphics and/or natural language. In this way it is possible to organize, understand and communicate the mathematical nature of actions previously carried out in the educational and social spheres.

2 Methodology

A qualitative approach is used to test the opportunities for understanding provided by the most concrete context (real situations) and the most abstract (graphic resources) of the EIEM (Alsina, 2019, 2020) when teaching repetition patterns to early childhood education students (4-6 years old). According to Maldonado (2018), this approach relies on the interpretation, description, analysis and comprehension of qualitative information obtained through recordings, observations, interview, etc. In keeping with this contribution, our design facilitates a descriptive and interpretative analysis that allows to show through the representation in mathematics of the students, the results obtained in a longitudinal way at the informal level, specifically in the context of real situations by comparing the results collected at the formal level (graphical resources).

2.1 Design and procedure

As indicated, the activities within the context of real situations and graphical resources that make up the IEEM have been selected. The six proposals have been submitted to experts who have assessed: a) didactic aspects, b) organizational aspects, c) methodological aspects, and d) pedagogic aspects of the teaching process. This procedure, along with the reflexive practice developed after each session, has favored the articulation of continuous and retrospective analyzes that inform the design and facilitate its improvement longitudinally. This procedure can be complemented by reading Acosta and Alsina (2020), who validated and applied an itinerary of repetition patterns with 3-year-old school children.

The implementation has been carried out in a longitudinal way, with 24 Spanish students all belonging to the same class of a public school. The sample consists of 12 boys and 12 girls. The average age of the sample was 4.8 years and 5.8 years old for the two school years. This group was selected because of the ease of access; because of the continuity and longitudinal follow-up of the tutoring teacher; and because it is considered a school with low enrollment rate in preschool grades.

Table 1

Proposals developed according to the teaching context

4-5 years old	
Real situations	A1. Google Maps is used on the whiteboard to display different streets in our city in search of mathematical patterns. Through questions, children are told to look at the facades of houses, buildings, and shops. Once the patterns have been identified, they are reproduced together using colored cardboards.
	A2. An image of a garden is shown, and students are invited to describe how the bushes are placed. Students are asked if they believe the bushes follow a sequence and are proposed to recreate a series with play dough.
Graphic Resources	Through a pre-written task designed with different types of awnings, children are told to expand the series.
5-6 years old	
Real situations	A1. Students are presented a basket with socks and sweaters with various designs; chess set, toy piano; card set, pictures of tiles; pieces of fabric with skin drawings of some animals, pictures of awnings…and they are invited to “hunt” and identify the patterns present in the objects inside the basket.
	A2. Walk through the school yard to get photographically the patterns existing in this educational space. Next, the students are presented with the challenge of playing some of the series found in a role.
Graphic Resources	Through previously designed forms, students are told to observe, identify, analyze, and read the series proposed to recognize the elements that make up the minimum unit of the pattern and to complete the series.

The activity is carried out in a small group (12 boys and girls) to ease individualized attention and the collection of specific and personalized evidence. The distribution of participants is done at random and the two subgroups of 12 students are maintained throughout the activity. Thus, in a longitudinal way, a total of eight direct intervention sessions are allocated for real situations and four sessions for graphical resources, each lasting 50 minutes. It should be mentioned that informed consent was obtained from all families before the intervention.

The sessions were divided into three phases: a) introduction of the proposal, b) interaction and development, and c) representation and reflection. It is important to emphasize that in the final phase the students represent the pattern that they identified in the activity without having the model in front of them. The role of the teacher is to guide and encourage learning through intentional questions (NCTM, 2014) that promote knowledge and share it with the peer group. Questions that do not involve reasoning or argumentation by students and that are answered with a "yes" or "no" should be avoided.

2.2 Data collection

Data collection includes three tools: I) ethnographic methodological schemes of participant observation using the field diary as a tool to record spontaneous expressions of children during the performance of tasks; II) pedagogical documentation through the audiovisual record, fixed and mobile, of all the sessions; and III) written productions in drawing format of all the representations of students as a sign of the formalization of the knowledge acquired.

Kawulich (2006) considers participating observation as a skill that enables researchers to reflect on and learn about proposals that are developed with participants in a natural context, using observation and active participation as facilitating tools for direct, non-interference interaction. On the other hand, the pedagogical documentation adopts a reflexive character that gives voice to the thought of the child, recognizing the observer as an active agent that co-constructs meaning in a reflexive, active and reciprocal way with the purpose of creating a plural and transformative space (Mitchelmore, 2018). We cannot ignore that verbal and non-verbal expressions are key to interpreting the knowledge and skills of younger schoolchildren (Björklund *et al.*, 2020).

2.3 Analysis of the data

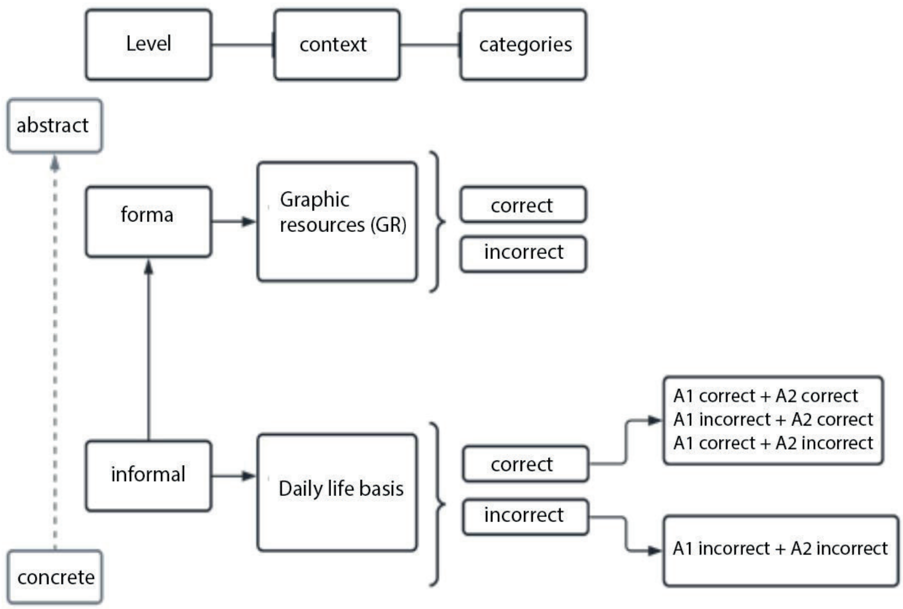
Children's drawings have been categorized according to the diagram below with the intention of eliminating the bias generated by a hierarchical presence of proposals according to the model proposed by the IEEM (Alsina, 2019, 2020). It is considered the "correct" category when the representation is error-free, and "incorrect" when the production is error-free in its structure.

Based on the information shown in Figure 1, a quantitative analysis was performed describing the categorical variables according to the number and percentage of cases in each category.

To support the latter, the audiovisual evidence is transcribed and discussed doing an interpretative analysis of the discourse. This analysis of the most relevant parts allows to review the relationships between texts and reality by presenting the discourse used by the child, his/her point of origin, how it flows, and what accompanies it (Leeuwen, 2008). This information is contrasted with the quantitative data, with the recorded field notes and with the written productions of the boys and girls to also show the role of the teacher.

Figure 1

Flowchart with the categorization process of the representations obtained



3 Results

Considering the aim of the study, the results obtained longitudinally are analyzed in contexts of real situations and graphic resources, with the intention of checking how the teaching context influences the understanding and representation of tasks with repetition patterns.

Table 2

Results achieved in 4-5 years old students

Real Situations	Frequency	Percent	Pct. Valid
Correct	17	70,8	85,0
Incorrect	3	12,5	15,0
Total Valid	20	83,3	100
Invalid	4	16,7	
Total	24	100	
Graphics Resources	**Frequency**	**Percent**	**Pct. Valid**
Correct	11	45,8	45,8
Incorrect	13	54,2	54,2
Total Valid	24	100	100
Invalid	0	0,0	
Total	**24**	**100**	

As shown in Table 2, 85% of valid cases represented correctly the pattern identified in the activities carried out in the context of real situations, compared with 15% who failed to perform the task successfully. However, there is a significant increase in errors in the context of graphical resources, located in 54.2%. The degree of success in this context is only 45.8%.

Below are the results corresponding to 5-6-year-old students.

Table 3

Results obtained for 5–6-year-old students

Real Situations	Frequency	Percent	Pct. Valid
Correct	23	95,8	100,0
Incorrect	0	0,0	0,0
Total Valid	23	95,8	100,0
Invalid	1	4,2	
Total	**24**	**100**	
Graphics Resources	**Frequency**	**Percent**	**Pct. Valid**
Correct	16	66,7	69,6
Incorrect	7	29,2	30,4
Total Valid	23	95,8	100,0
Invalid	1	4,2	
Total	**24**	**100**	

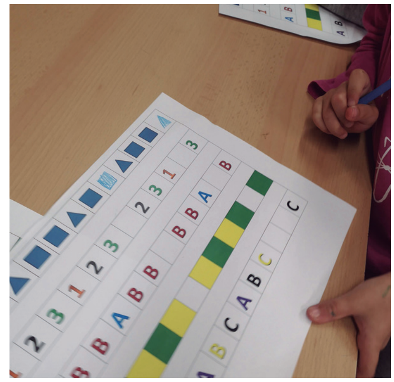
According to the information shown in Table 3, 100% of valid cases represented without errors the patterns identified in the context of real situations, while only 69.6% did so in the context of graphical resources. We observed that incorrect representations decreased by 23.8 % compared to previous year.

In Table 4 we show some examples of the implementation. One is selected for each context and age due to the space.

Table 4

Evidence of the conduction of activities in each context according to the age

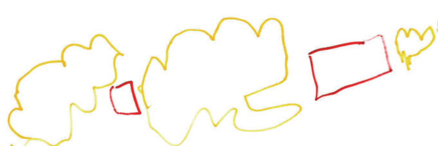
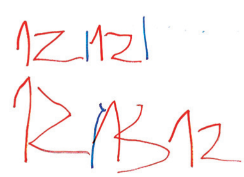
Context	4-5 years	5-6 years
Real Situations		
	Plays series Using colored cardboards (A1)	**Discovering series with yard elements (A2)**

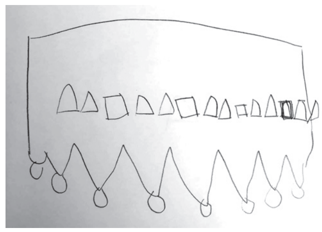
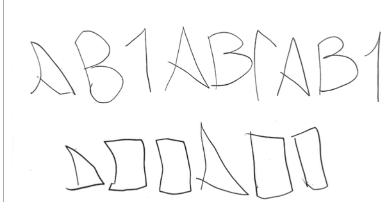
Context	4-5 years	5-6 years
Graphics Resources		
	Expand series	Complete missing elements of the series

Below are some examples of representations and transcripts of dialogs obtained during the pedagogical implementation to illustrate the role of the teacher as a generator and promoter of learning.

Table 5

Examples of correct representations obtained in each context according to the age

Context	4-5 years	5-6 years
Real Situations	 **Teacher:** What does your drawing represent? **Student:** The awning we saw. **Teacher:** Why do you use two colors? **Student:** Because it was yellow-brown, yellow-brown. **Teacher:** And the awning that had white and blue stripes, is it the same as this one? **Student:** Yes, because it also has two different colors.	 **Student:** I painted 1Z and blue line, 1Z and blue line. **Teacher:** What object of the basket of treasures have you represented? **Student:** The pink-lilac-white striped jumper. **Teacher:** Can you explain why you have a 1, a Z and a blue line? **Student:** Because pink, lilac and white are different colors. **Teacher:** So, you have assigned number 1 to pink and Z to lilac? **Student:** Yes, and the blue line to white.

Context	4-5 years	5-6 years
Graphics Resources	 **Student:** Look, I painted triangles and squares **Teacher:** Can you explain what does your drawing represent? **Student:** These are two triangles and a square, two triangles and a square and so on to infinity. **Teacher:** Why did you paint two triangles and a square? **Student:** Because my birthday crown was like that.	 **Teacher:** Can you explain your drawing? **Student:** On the card, there was a pattern with letters and also numbers. **Teacher:** Then, have you done the same of the file? **Student:** No, because I had the C and I used number one. **Teacher:** The pattern of the file was ABCABCABC, yours is AB1AB1AB1, are they the same or different? **Student:** They are different. **Teacher:** But if you look at them, they have three different elements (ABC) and (AB1). Then we could consider them as the same because they have the same repetition structure.

From the examples shown in Table 5, it can be seen how the student justification is more elaborated and consolidated in the context of real situations than in the context of graphic resources. In the same way, it is seen how the teacher, through good questions, i.e., open questions and using a precise mathematical language, motivates the students to communicate, justify and reason their answers. This scenario enables a constructive discussion from a parallel perspective lived by another participant, that favors enrichment and conceptualization in those students who have not succeeded in the task (Vygotsky, 2004).

4 Discussion and conclusions

This study investigated how the teaching context influences the performance of tasks with repetition patterns. To this end, students' written productions were analyzed to determine whether they were able to correctly represent repetition patterns in the most concrete context (real situations) and in the most abstract (graphic resources) of a previously designed and validated pattern teaching process. Based on this analysis, a positive difference of 32.9% of real situations versus graphic resources was identified in the 4-5-year-old students; in the 5-6-year-old, although the difference between the two contexts decreased slightly, it was still over 30%.

If the results are analyzed longitudinally, 15% of 4-5-year-old students have incorrectly represented patterns in tasks posed from real-world situations, while the percentage of incorrect pattern drops to 0% in 5–6-year-old students. In the context of graphical resources, incorrect representations decreased by 23.8%, 54.2% for 4-5-year-old students and 30.4% for 5–6-year-old students. However, despite this decrease, participants generally show difficulty in representing the pattern without prior interaction or manipulation of specific elements that make up the repetition. In other words, the data obtained have shown that understanding is more accurate in contexts where teaching is prioritized from informal situations of exploration of a daily environment close to boys and girls, where it is easier to establish relationships with their previous knowledge. In this sense, Castro and Castro (2016) point out that they learn through concrete experiences with materials and through intentional and previously planned recreational interactions. Likewise, Zhong and Xia (2020) say that children need opportunities for exploration, manipulation, and experimentation to promote learning from a playful and concrete perspective.

Focused on representations of patterns, we agree with Alsina (2016) when mentioning that the representation of mathematical ideas and procedures is an essential process for learning, and therefore, if there is no representation there is no understanding, and thus there can be no learning of mathematics. Therefore, it can be assumed that, from an early age, children must represent to learn mathematics and thus be able to organize, understand and communicate the mathematical nature of the actions previously carried out at the educational and social level using signs, graphics and/or natural language. This is the main reason why we have used representations in mathematics as an interconnected process that allows to: a) concretely embody the knowledge and procedures of children about repeating patterns; b) assess progress in understanding these patterns; and c) rebalance the process of teaching patterns through the design of contextualized tasks that encourage and extend learning.

From this perspective, Laski and Siegler (2014) show that concrete learning materials are only effective to the extent that the activities designed are aligned with the desired mental representation process. It is for this reason that educational proposals must be related with the aim of increasing the codification of the structural characteristics that make up the pattern to facilitate representation. Carruthers and Worthington (2005) conclude that when teachers encourage children from 3 to 8 years old to play their mathematical ideas on paper, they encourage an understanding of abstract symbolism.

This study has provided relevant data showing how the degree of success of understanding through representation is conditioned by the abstraction level of the context in which the proposal is presented. In this regard, it is considered necessary to plan and structure tasks that include different teaching contexts, in order to offer an educational intervention respectful to the needs of the students, in which it is essential to encourage the use of concrete and informal contexts that allow progress toward the generalization and formalization of knowledge, avoiding patterns that exclusively use paper and pencil. This approach requires, on the one hand, disciplinary knowledge about what is wanted to teach (NCTM, 2014; Pincheira and Alsina, 2021); and, on the other, didactic and methodological skills to deal with a particular concept or procedure from different teaching contexts (Alsina, 2022). Villalpando *et al*. (2020) point out that the professors must transfer the official program into a real setting that allows to give meaning to the teaching practice, in order to bring the academic contents to the students in a reflective, competent and experiential way.

At this point, we think that educational situations are context-sensitive by showing that the success of the representation has been closely related to the understanding of the pattern, and that understanding has been most successful at the most concrete level of the EIEM, in which the teaching of mathematical content is prioritized for situations that are real or close to children. It has also been demonstrated that shared knowledge is promoted, generated, and consolidated by using good questions. For this reason, we encourage teachers to support the teaching of repetition patterns from a dialogic and multimodal vision that includes various educational scenarios that move progressively from concrete to abstract contexts. Therefore, our aim is that these real experiences accompany, through reflection, future teaching action (Radford and Sabena, 2015) and that our conclusions could be a source of inspiration, without being directly generalized to other realities, since the small number of our sample is a limitation. In this regard, we also assume as a limitation the deferred use, through images, of real situations in the first stage of the didactic process, since it may have influenced the responses of the students, and we cannot know for sure whether errors would have been reduced with a live deployment. As future lines of research, we propose to continue to demonstrate how the other contexts of the EIEM influence the teaching of repetition patterns, analyzing the relationship established between the mathematical knowledge of the students and the ability to justify and argue their responses.

Acknowledgements

This research was supported by the Spanish Ministry of Education, Culture and Sports under the Grant for University Teacher Training (FPU16-01856). The authors are also by the help provided by Pericot school in Girona and to Mireia Moran, teacher of the participating boys and girls.

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